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The Theory of Wave Transformation in Ring Dielectric Waveguide

V. B. Kazanskiy, I. V. Kocarevskaya

Kharkiv State University
Dept. of theoretical radiophys.
4 Sq. Svobody, Kharkiv 310077, Ukraine

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Eigenwaves and energy processes in ring dielectric waveguide based on rigorous solution of propagation problem have been studied. Diffraction spectrum of radiated waves, energy fluxes and transparency of periodic border for TE-, TM-, and hybrid waves have been found.

Resonance conditions of transformation of guide wave into volume waves of free space have been determined. The effectivity of the transformation depends on polarization and is greater for TM_{0p}-waves. It is shown that propagation of space harmonic and corresponding energy flux do not coincide. That leads to longitudinal shift of local reflective point (Goos-Hänchen's shift) of plane wave corresponding to space harmonic. Its analytical estimation has been given and effective radius taking into account field leakage outside waveguide borders has been presented.

Space distribution of energy flows under various regimes of propagation of electromagnetic waves of arbitrary types in circular metal-dielectric ring waveguide has been studied. Rigorous conditions of effective transformation of slow waves of dielectric channel into body waves have been found. Estimation of Goos-Hänchen's displacement for waveguide with impedance (ring) surface is given.

Circle dielectrical waveguide is investigated of radius R and permittivity ε_0 with periodic grating of infinite thin and perfectly conducting rings with a period l and a gap d on its surface. External medium has permittivity ε_1 ($\varepsilon_0 < \varepsilon_1$) (Fig. 1). The waves of this structure for arbitrary azimuthal distribution of electromagnetic field can be described by longitudinal components of electric Π_z^{el} and magnetic Π_z^{mg} potentials:

$$\begin{Bmatrix}\Pi_{zj}^{el}\\ \Pi_{zj}^{mg}\end{Bmatrix} = \sum_{m=-\infty}^{\infty}\begin{Bmatrix}A_{mj}\\ B_{mj}\end{Bmatrix}\times Z_{nj}(\gamma_m^j r)\exp[i(h_m z - n\phi)], \qquad (1)$$

where $Z_{nj}(x)$ is cylindrical function equal to $J_n(\gamma_m^1 r)$ in internal range of waveguide ($j=1$) and $H_n^{(1)}(\gamma_m^0 r)$ in external space; $h_m = h_0 + 2\pi m/l$, phase constant of m-th diffraction harmonic: $\gamma_m^j = \sqrt{k^2\varepsilon_j - h_m^2}$ – radial constant and $\beta = v/c = k/h_0$ – the slowdown coefficient of waveguide.

The propagation constant h_0 and the relationship between coefficients A_{mj} and B_{mj} are determined from the solution of propagation problem by Rieman-Gilbert's method. The solution is represented in the form of the system of the connected uniform algebraic equations of Fredholm's type. Equating to zero the system's determinant, the slowdown coefficient β of arbitrary types of waves is calculated through functions $W_{mj}^{\sigma}(\cos(\pi d/l))$ [1].

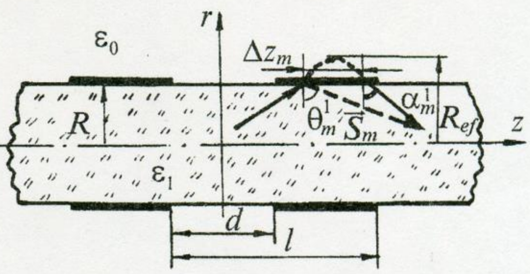

Fig. 1. *Structure of study*

Here we will study regimes of slow waves in dielectric ring waveguide ($1/\sqrt{\varepsilon_1} < \beta < 1$) for $d/l = 0.5$ in resonance range. According to the ray analogy [2], the direction of propagation of diffrection harmonics will be determined through the angles:

$$\alpha_m^j = \arcsin\left[\left(1/\sqrt{\varepsilon_j}\right)\left(1/\beta + m/\kappa\right)\right], \qquad (2)$$

which are measured from the normal; index $j=1$ corresponds to reflected field and $j=0$ – to transmission field. These angles define the conditions $\kappa/\left(\kappa\sqrt{\varepsilon_0}-m\right)<\beta$ of transformation of slow wave in dielectric waveguide into cylindrical wave of free space.

The diffractional spectrum of quasi open waveguide has some characteristic properties. The first, by virtue of the specification of functions $W_{mj}^{\sigma}(0)$ the total field includes only space harmonics with numbers $m=0$, $m=2k$ and $m=-2k+1$. The second, in view of (2) the maximal absolute values of indexes of positive $M^{+}_{\max j}$ and negative $M^{-}_{\max j}$ harmonics corresponding to angles $\alpha^{j}_{M^{\pm}}\to\pm\pi/2$ (the creeping regime) are determined by inequalities $0\le M^{\pm}_{\max j}<\kappa\left[\sqrt{\varepsilon_j}\mp 1/\beta\right]$. Hence, the diffraction spectrum consists predominantly of negative space harmonics, and if $\varepsilon_0=1$, any positive harmonic will not propagate in free space. When $m=m_S=-\kappa/\beta$ the effect of space resonance takes place [3], $\alpha^{0}_{m_S}=\alpha^{1}_{m_S}=0$, and harmonics with numbers (m_S-k) are mutually reciprocally compensated. The waveguide radiates for m_S-th harmonic in the plane, which is perpendicular to axis, practically, without losses to reflection into interior range.

Transformation effectiveness of slow wave into body wave is determined by the comparative distribution of density of the energy flux $\vec{S}=\left[\vec{E},\vec{H}\right]$ in a periodic border's vicinity. Calculation of Umov-Poynting's vector based on rigorous theory is concerned with great expenditure of computer time [1]. In order to get its high-quality estimation we will use the ray analogy [1] and method of "effective waveguide's radius" with correction obtained from rigorous theory.

For symmetrical H_{0p}- and E_{0p}- waves ($\vec{S}=\vec{e}_r S_r+\vec{e}_z S_z$) concentration of field energy of fundamental slow wave in the vicinity of $R(O^{+})$ depends on the transparency of periodic border $\rho_\nu\sim 1/\left|\Delta S^{\nu}_r\right|$ ($\nu=el,mg$), where the change of module of radial component of vector $\vec{S}$ is proportional to the jump of longitudinal component H_z (H_{0p}- waves, $\nu=mg$) or azimuthal component H_φ (E_{0p}- wave, $\nu=el$) at the boundary $r=R$. Taking into account that for $m=0$ $\mathrm{Re}\left(\gamma^{1}_{0}\right)=0$ and $\mathrm{Im}\left(\gamma^{1}_{0}\right)>0$ the rigorous solution turns to the following expressions:

$$\left|\Delta S^{mg}_r\right|=k\gamma^{1}_{0}a_0^2\left|\frac{J_0\left(\gamma^{1}_{0}R\right)}{J_1\left(\gamma^{1}_{0}R\right)}+\frac{\left|\gamma^{0}_{0}\right|K_0\left(\left|\gamma^{0}_{0}\right|R\right)}{\gamma^{1}_{0}K_1\left(\left|\gamma^{0}_{0}\right|R\right)}\right|\sim 2(1-\mu),$$

$$\left|\Delta S^{el}_r\right|=k\gamma^{1}_{0}a_0^2\left|\varepsilon_1\frac{J_1\left(\gamma^{1}_{0}R\right)}{J_0\left(\gamma^{1}_{0}R\right)}+\frac{\varepsilon_0\gamma^{1}_{0}K_1\left(\left|\gamma^{0}_{0}\right|R\right)}{\left|\gamma^{0}_{0}\right|K_0\left(\left|\gamma^{0}_{0}\right|R\right)}\right|^{-1}\sim\frac{1-\chi}{\varepsilon_1+\varepsilon_0},\tag{3}$$

where a_0 is the amplitude coefficient of zero spectral component proportional to ratio $(\beta/\kappa)^2$, and does not dependent on wave's type; χ and μ are small parameters in the Rieman-Gillbert's problem [1]. From (3) it follows that when $d=0.5l$ and $\varepsilon_1>\varepsilon_0\ge 1$ the border's transparency for E_{0p}-waves is greater than for H_{0p}-waves, and the character of dependence of ρ_ν on polarization ramains for quick waves ($\beta>1$) too (Fig. 2). For azimuth-inhomogeneous hybrid HE_{np}- and EH_{np}- waves it is very difficult to state relationship between ρ_{he} and ρ_{eh} (Fig. 3). However, analysis of numerical experiment shows that the primary type of polarization is a decisive factor.

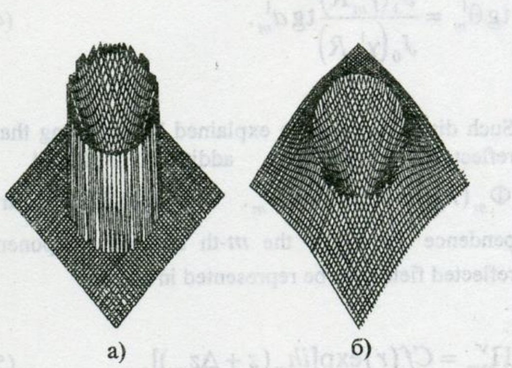

Fig. 2. *Module of radiate flux of energy at plane* $r-\varphi$ *(viewing angle:* $-45°$, $-45°$, $60°$; $\varepsilon_1=2.08$; $\varepsilon_0=1$; $l/R=0.1401$*) for*

a) H_{01}*-wave:* $kR=1.5\pi$; $1/\beta=1.187$; $\kappa=0.1051$;

b) E_{01}*-wave:* $kR=1.7\pi$; $1/\beta=0.523$; $\kappa=0.1191$

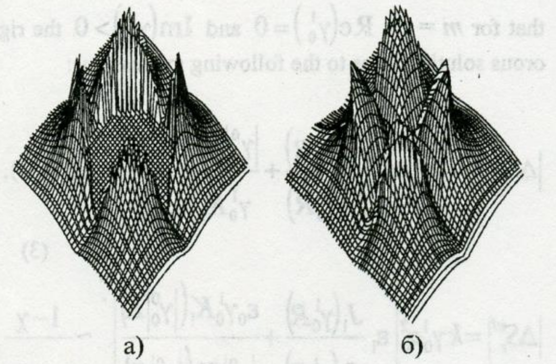

Fig. 3. *Module of radiate flux of energy at plane* $r-\varphi$ *(viewing angle:* $-45°$, $-45°$, $60°$; $\varepsilon_1 = 2.08$; $\varepsilon_0 = 1$; $l/R = 0.1401$*) for*
a) HE_{11}*-wave:* $kR = 0.9\pi$; $1/\beta = 1.277$; $\kappa = 0.083$;
b) EH_{01}*-wave:* $kR = 1.4\pi$; $1/\beta = 0.785$; $\kappa = 0.0981$

Proceeding from the strict estimation of the radial component jump $\bar{S}_m$ at $\beta < 1$ it is easy to obtain that the reflection angle of m-th diffraction harmonic a^1_m from surface $r = R$ and the direction of energy flux of m-th spectral field component θ^1_m do not coincide and are relacted by equation:

$$\operatorname{tg}\theta^1_m = \frac{J_1(\gamma^1_m R)}{J_0(\gamma^1_m R)} \operatorname{tg} a^1_m. \qquad (4)$$

Such distinction can be explained by assuming that the reflected wave gets additional phased shift $\Phi_m(h_m) = k\sqrt{\varepsilon_1}\sin\theta^1_m$. With the limit linear dependence $\Phi_m(h_m)$ the m-th spectral component of reflected field may be represented in a form:

$$\Pi^{\nu}_{zm} = Cf(r)\exp[ih_m(z + \Delta z_m)], \qquad (5)$$

where $\Delta z_m = \partial\Phi_m / \partial h_m$ is considered as longitudinal shift of the reflection point of plane wave. This shift is known in literature as Goos-Hänchen displacement [4] and reflects the real leakage of slow wave field into external optically less dense medium. In case studied with account (4) we get:

$$\Delta z_m = \left(h_m / k\sqrt{\varepsilon_1}\right)^2 \left(1 - \gamma^1_m R\right) / k\sqrt{\varepsilon_1}. \qquad (6)$$

From (1) it follows that at $r > R$ the fields of slow diffraction harmonics in the waveguide under investigation attenuate exponentially with the growth of r and, therefore, it is practically located in the range of $R_{ef}(m)$ (Fig. 1). $R_{ef}(m)$ can be expressed through the longitudinal shift Δz_m and diffraction angle α^1_m:

$$R_{ef}(m) = R\left[1 + \Delta z_m \left|\operatorname{ctg}\alpha^1_m\right| / 2R\right]. \qquad (7)$$

Thus, using value of β, obtained from rigorous solution [2] for m-th space harmonic, the original structure may be replaced by regular circle waveguide of radius $R_{ef}(m)$ with $\varepsilon = \varepsilon_1$. It permits to get simple analytical expressions for ρ_ν:

$$\rho_{mg} = \frac{\kappa^2 R_{ef}}{k\beta^2 \mu_{0p}} \left| \frac{J_1(\mu_{0p} R / R_{ef})}{J_0(\mu_{0p} R / R_{ef})} \right|,$$
$$\rho_{el} = \frac{\kappa^2 R_{ef}}{k\beta^2 \nu_{0p}} \varepsilon_1 \left| \frac{J_1(\nu_{0p} R / R_{ef})}{J_0(\nu_{0p} R / R_{ef})} \right|, \qquad (8)$$

where $R_{ef} = R_{ef}(0)$, ν_{0p} and μ_{0p} are p-th roots of $J_0(z)$ and $J_1(z)$, respectively.

The expressions (8) indicate to resonance character of dependence $\rho_\nu(\beta)$. As ratio μ_{0p}/ν_{0p} is greater than 1 and diminishes with the growth of radial index, and $R_{ef} > R$ (7), then at certain set of parameters a situation $\left(\mu_{01} R / R_{eff} \approx \nu_{01}\right)$ may arise when border's transparency for H_{0p}-wave begins to increase abruptly. This leads the electric field strength increase on rings and hence to resonance radiation of body waves into free space. For E_{01}-wave such effect is impossible in principal, but for E_{0p}-wave ($p \geq 2$) the growth of ρ_{el} can be achieved if $R/R_{ef} \approx \nu_{0[p-q]}/\nu_{0p}$. It should point out the presence of conditions for resonance reflection of higher waves ($p \geq 2$) of both polarizations when $\rho_\nu \to 0$, and the efficiency of excitation of the diffraction spectrum suddenly decreases.

The phenomena of space resonance may be described within the bounds of the offered approach. Really, for $\alpha^1_{m_s} = 0 \quad R_{ef}(m_s) \to \infty$ and border's transparency for the resonance m_s -th harmonic is estimated as:

$$\rho_{mg} = \frac{m_s^2}{k\mu_{0p}} \lim_{\alpha_{m_s} \to 0} R_{ef}(m_s) J_1\left[\mu_{0p} R / R_{ef}(m_s)\right] = \frac{m_s^2 R}{2k},$$

$$\rho_{el} = \frac{\varepsilon_1 m_s^2}{k\nu_{0p}} \lim_{\alpha_{m_s} \to 0} R_{ef}(m_s) J_1\left[\nu_{0p} R / R_{ef}(m_s)\right] = \frac{\varepsilon_1 m_s^2 R}{2k} \quad (9)$$

As follows from (8), (9), periodic border is polarizationally sensitive because $\rho_{el}/\rho_{mg} \sim \varepsilon_1$. Therefore, the efficiency of space harmonics excitation and, hence, the transformation of slow waves into body waves is higher for E_{0p} -waves. Note, that the higher is the permittivity ε_1 of the dielectric channel the greater this pecularity displays.

Thus, the qualitative estimations obtained permit to conclude about the specific character of energy processes in circle periodic ring waveguide. The results of approximate method of "effective waveguide" are in good agreement with the conclusions of strict theory and indicate to particularly pronounced polarized dependence of transformation of slow wave of dielectric waveguide into body wave of free space.

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Теория преобразования волн в кольцевом диэлектрическом волноводе

В. Б. Казанский, И. В. Коцаревская

Исследуются собственные волны и энергетические процессы в кольцевом диэлектрическом волноводе на основе строгого решения задачи распространения. Определены дифракционный спектр излучаемых волн, потоки энергии и прозрачность периодической границы для ТЕ-, ТМ- и гибридных волн.

Выявлены резонансные условия преобразования волноводных волн в волны свободного пространства. Эффективность преобразования зависит от поляризации и оказалась выше для TM_{0p}-волн. Показано, что направления распространения пространственных гармоник и соответствующего им потока энергии не совпадают. Это приводит к продольному сдвигу локальной точки отражения (смещение Гуса-Хенхена) плоской волны соответствующей пространственной гармоники. Дана его аналитическая оценка и определен эффективный радиус волновода, учитывающий просачивание поля за его границы.

Теорія перетворення хвиль у кільцевому діелектричному хвилеводі

В. Б. Казанський, І. В. Коцаревська

Досліджуються власні хвилі та енергетичні процеси у кільцевому діелектричному хвилеводі на підставі точного розв'язання задачі розповсюдження. Визначено дифракційний спектр випромінюваних хвиль, потоки енергії та прозорість періодичної межі для ТЕ-, ТМ- і гібридних хвиль.

Виявлено резонансні умови перетворення хвиль у хвилеводі на хвилі вільного простору. Ефективність перетворення залежить від поляризації і виявилась вищою для TM_{0p}-хвиль. Показано, що напрямки розповсюдження просторових гармонік і відповідного до них потока енергії не співпадають. Це призводить до повздовжнього зрушення локальної точки відбиття (зсуву Гуса-Хенхена) плоскої хвилі відповідної просторової гармоніки. Надано його аналітичну оцінку та визначено ефективний радіус хвилевода, який враховує проникнення поля за його межі.