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ПОШИРЕННЯ, ДИФРАКЦІЯ І РОЗСІЯННЯ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ

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A. V. GRIBOVSKY
Institute of Radio Astronomy,
National Academy of Sciences of Ukraine,
4, Mystetstv St., Kharkiv, 61002, Ukraine
E-mail: grib@rian.kharkov.ua

A FABRY-PEROT METARESONATOR SUPPORTING TRAPPED-MODE RESONANCES

Purpose: *Investigation of the electrodynamic properties of a Fabry-Perot metaresonator formed by two parallel perfectly conducting, two-dimensionally periodic, two-element screens of finite thickness with rectangular holes. The resonator is excited by a plane linearly polarized electromagnetic wave. The basic cell of each of the screens used as the metaresonator mirrors contains two lengths of rectangular waveguides of different transverse sections.*
Design/methodology/approach: *An operator method for solving the 3D problems of electromagnetic wave diffraction by multi-element two-dimensionally periodic structures is used in the study. The computation algorithm uses the partial domain technique and the method of generalized scattering matrices.*
Findings: *As follows from the results of the numerical modeling made, the magnitude of the plane wave reflected from the metaresonator turns to zero at fixed frequencies lying below the cutoff frequencies for the rectangular waveguide sections embedded in the resonator mirrors. The effect of the total electromagnetic wave transmission through the metaresonator at the first lower frequency is characterized by a strong localization of the electromagnetic field in the resonator volume. The reason is excitation of the metaresonator by the exponentially descending field penetrating inside the resonator through the evanescent holes at the resonance frequency. The second low-frequency resonance of the total electromagnetic wave transmission through the metaresonator is associated with the trapped-mode resonance, which is observed in multielement two-dimensionally periodic structures. This case is characterized by a strong localization of the electromagnetic field from both sides near the metaresonator mirror surfaces.*
Conclusions: *The unique electrodynamic properties of the metaresonator can find application in the devices for measuring the electrophysical parameters of composite materials with high losses. The effect of strong localization of the electromagnetic field both in the resonator volume and near the mirror surfaces can be used for monitoring the gaseous substances in crowded places.*

Key words: *two-dimensionally periodic screen, rectangular waveguide, Fabry-Perot metaresonator, reflection factor, evanescent waveguide, trapped-mode resonance*

1. Introduction

In view of the today's trend in electronics to use yet shorter electromagnetic waves, including terahertz frequencies, a demand arises for the development of appropriate electronic components. Among the most sought-after elements for this purpose, one is the Fabry-Perot resonator, which can be used as a highly selective unit with frequency-dependent parameters.

Recently, many researchers analyze the possibility of using the Fabry-Perot resonators in antenna engineering. For example, paper [1] presents a new antenna array of circular polarization based on the Fabry-Perot resonator, which provides a wider amplification band as against the conventional antennas. A microstrip resonator antenna with dual circular polarization built around a dual-band polarized Fabry-Perot analyzer is studied in paper [2]. The authors of review [3] consider the antennas with metaresonator, which resonant properties of metamaterials are used to reduce the dimensions of radiators and designs of multi-band antennas.

It seems of interest to investigate the electrodynamic properties of the Fabry-Perot resonators in which metal screens of a finite thickness perforated by holes of complex geometry are used as the "mirrors". It will be observed that the electrodynamic characteristics of the Fabry-Perot resonators with rectangular and coaxial-sector holes have been theoretically and experimentally investigated in detail in papers [4–7]. In particular, such two-layer structures based on the Fabry-Perot resonators with semitransparent mirrors have been shown to possess unique properties and can be applied in various fields of science and technology. This paper shows the results of pioneering theoretical investigations of a Fabry-Perot metaresonator formed by two metasurfaces, which support trapped-mode resonances.

2. Problem Formulation and Solution Technique

Consider a structure consisting of two identical, infinite within the $\{x, y\}$-planes, parallel, perfectly conducting, two-dimensionally periodic, two-element screens of a finite thickness h with rectangular holes. The mirrors spacing is equal to H and has been selected from the condition $H \geq \lambda/2$, with being the free space wavelength. Shown in Fig. 1 are the area fragment of the Fabry-Perot metaresonator mirror and its basic cell. The basic cell centers are located in the nodes of a rectangular mesh. The holes in the mirrors are treated as sections of rectangular waveguides with the transverse sections $a_1 \times b_1$ and $a_2 \times b_2$. The sizes of the transverse sections are taken such that the fundamental waveguide mode alone be propagational through these. The basic cell centers are located with the same periods in both screens, specifically, d_1 and d_2 along the x- and y-axis, respectively (see Fig. 1). At that, the periods are selected such that a single partial harmonic could propagate in free space.

Let a plane linearly polarized electromagnetic wave of the unit amplitude, $E_y = \exp(-ikz)$, be incident upon the structure under consideration from the $z > 0$ half-space. The result of the plane electromagnetic wave scattering by the metaresonator is excitation of a discrete set of spatial harmonics in free space, which includes one propagational wave and an infinite number of surface waves propagating along the metaresonator mirror apertures from both sides. The transverse component of the reflected wave electric field is shown in the form of an expansion in the full set of orthonormal vectorial spatial *TE*- and *TM*-harmonics, viz.

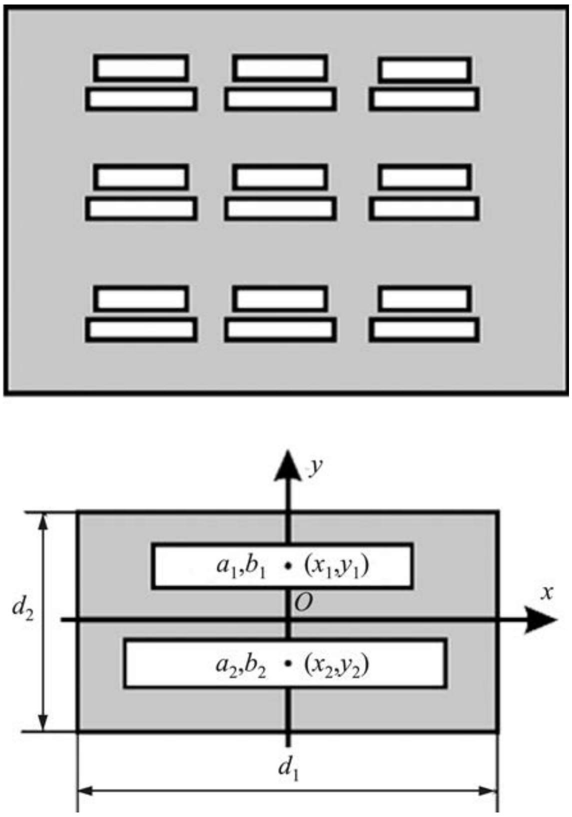

Fig. 1. A fragment of the metaresonator mirror and its basic cell

$$\vec{E}_t^r(x,y,z) = \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} b_{qs}^{(1)}\vec{\Psi}_{qs}^{(1)} e^{i\Gamma_{qs}z} + \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} b_{qs}^{(2)}\vec{\Psi}_{qs}^{(2)} e^{i\Gamma_{qs}z}, \qquad (1)$$

with $z > 0$.

Here, $b_{qs}^{(1)}$ and $b_{qs}^{(2)}$ are unknown amplitudes of the spatial *TE*- and *TM*-harmonics, respectively; the orthonormal vectorial spatial harmonics $\vec{\Psi}_{qs}^{(l)}$ are determined with the formulas

$$\vec{\Psi}_{qs}^{(l)} = \exp\left\{i\left(\kappa_x x + \kappa_y y\right)\right\}\frac{1}{\sqrt{S_0}\,\kappa_r}\begin{cases}\kappa_y\vec{e}_x - \kappa_x\vec{e}_y & l = 1,\\ \kappa_x\vec{e}_x + \kappa_y\vec{e}_y & l = 2,\end{cases}$$

where $\kappa_x = k\sin\theta_0\sin\varphi_0 - \dfrac{2\pi q}{d_1}$ and $\kappa_y = k\sin\theta_0\cos\varphi_0 - \dfrac{2\pi s}{d_2} + \dfrac{2\pi q}{d_1\tan\chi}$, with $k = 2\pi/\lambda$ being the wave number, $S_0 = d_1 d_2$ is the transverse section area of the basic cell, $\kappa_r = \sqrt{\kappa_x^2 + \kappa_y^2}$, and $\vec{e}_x$ and $\vec{e}_y$ are unit vectors in the Cartesian coordinate system

xOy; and $\Gamma_{qs}=\sqrt{k^2-\kappa_r^2}$, with $\Gamma_{00}=k\cos\theta_0$. The time dependence has been taken to be $e^{-i\omega t}$. Angle χ determines the geometry of the mesh in whose nodes the centers of the metaresonator mirror basic cells are located. With $\chi=90°$, the basic cell centers are located in the nodes of a rectangular mesh. The angles θ_0 and φ_0 are incidence angles of the plane wave in a spherical coordinate system. In the case under consideration, $\theta_0=\varphi_0=0$.

To numerically investigate the electrodynamic characteristics of the given structure, let us use the method of generalized matrices of scattering by two-dimensionally periodic multi-element screens of finite thickness with rectangular holes [8].

Consider the section of one period of the metaresonator mirror within the zOy-plane (see Fig. 2). According to the notation in Fig. 2, let us write a set of operator equations with respect to the unknown amplitudes of the spatial harmonics propagating and damping in free space and resonator volume, viz.

$$\begin{cases} B=Rq+TC, \\ A=Tq+RC, \\ C=E_H R E_H A, \\ D=TE_H A. \end{cases}$$

Here q is the incident wave amplitude, B stands for the vector of spatial harmonic amplitudes of the reflected field, A and C are the amplitudes of the spatial harmonics propagating within the metaresonator volume, and D denotes the vector of spatial harmonic amplitudes of the transmitted field.

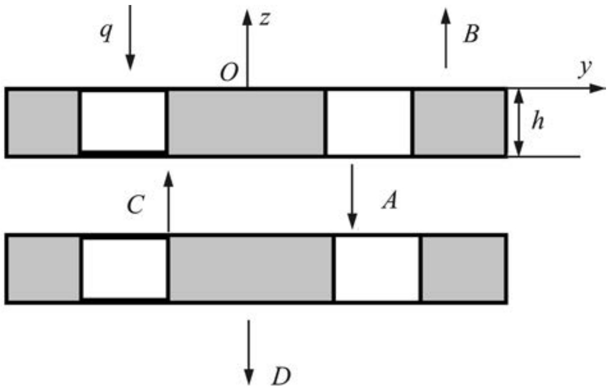

Fig. 2. A section of one period of the metaresonator mirror within the zOy-plane

The solution of the operator equation set is

$$B=(R+TE_H RE_H P^{-1}T)q,$$
$$A=P^{-1}Tq,$$
$$C=E_H RE_H P^{-1}Tq,$$
$$D=TE_H P^{-1}Tq,$$

where R and T are the operators of reflection and transmission of plane waves from/through the metasurfaces forming the resonator, E_H is the operator considering the phase incursion in the spatial harmonics on their travel between the resonator mirrors at the distance of H, and $P=(I-RE_H RE_H)$, with I being the unit operator.

3. Numerical results

The frequency selective properties of the metaresonator were investigated numerically for the case of normal incidence of a plane linearly polarized electromagnetic wave of unit amplitude upon the resonator. The polarization vector of the electromagnetic wave is oriented along the Oy-axis. This situation corresponds to the most efficient excitation of the electromagnetic field in the rectangular holes of the metaresonator mirrors, which are treated as sections of rectangular waveguides. The geometrical parameters of the metasurfaces forming the Fabry-Perot resonator were as follows: $a_1=14.1$ mm, $a_2=14.7$ mm, $b_1=2.95$ mm, $b_2=2.95$ mm, $d_1\times d_2=17.6\times17.6$ mm, $x_1=x_2=0$, $y_{1,2}=\pm5.88$ mm, and $h=2$ mm.

The critical frequencies of the rectangular waveguides in the metaresonator mirrors with the above parameters are $f_c^{(1)}=10.638$ GHz and $f_c^{(2)}=10.204$ GHz. The separation between the metaresonator mirrors is equal to $H=40$ mm. The geometrical parameters of the metaresonator mirrors have been taken the same as in paper [9]. In this latter one, the trapped-mode resonance has been theoretically investigated and experimentally validated to exist in the metasurfaces forming the metaresonator under consideration.

Shown in Fig. 3 is the frequency dependence of the reflection factor magnitude of the metaresonator excited by the plane wave.

As can be seen, the frequency dependence of the metaresonator reflection factor shows a resonance behavior. Specifically, the effect of total

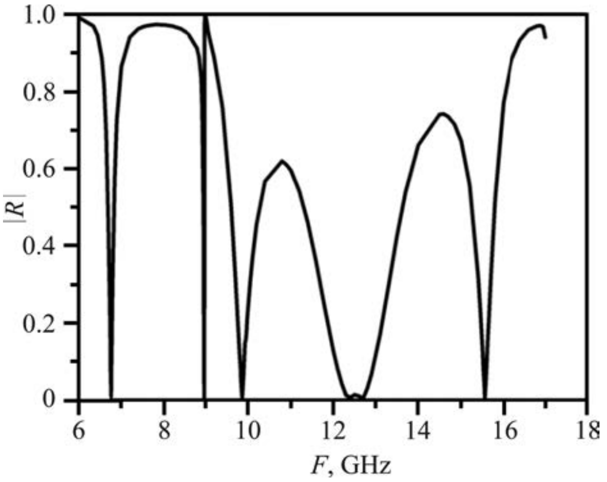

Fig. 3. Frequency dependence of the reflection factor magnitude of the plane wave excited metaresonator

transmission of the electromagnetic wave through the resonator at resonance frequencies is observed. In what follows we will be interested in those frequencies whose values are below the cutoff frequencies of the rectangular waveguides. Shown in Fig. 4 are magnitudes of the spatial harmonics propagating in the metaresonator volume depending on frequency.

The first (lowest frequency) resonance is observed at the frequency of $f = 6.76$ GHz ($\lambda = 44.38$ mm). It is associated with the metaresonator excitation by the exponentially decreasing electromagnetic field penetrating inside the metaresonator through the evanescent holes. This case is characterized by a strong localization of the electromagnetic field inside the metaresonator volume. To prove the effect, Fig. 5 shows the field magnitude in the metaresonator volume at the resonance frequency depending upon the mirror spacing. As can be seen, the adjacent peaks in the dependence are separated by approximately one half of the free space wavelength, and the field magnitude in the resonator more than twice exceeds the field amplitude of the incident plane wave. The reason, why the spacing between the metaresonator mirrors corresponding to the maxima of the curve in Fig. 5 is somewhat smaller than the respective integer number of the free space half waves, is associated with penetration of the electromagnetic field through the evanescent holes in the metaresonator mirrors.

The second low-frequency high-quality resonance of the electromagnetic wave total transmission through the metaresonator is observed at the frequency of $f = 8.969$ GHz ($\lambda = 33.45$ mm). This resonance is associated with the excitation of anti phase oscillations of the electromagnetic field in the waveguide channels of different transverse sections [8]. And, as was shown in paper [10], in this case the surface *TM*-harmonics of high amplitudes are excited propagating pairwise towards one another along the entire surface of the metaresonator mirrors from both sides. This is the trapped-mode resonance. Shown in Fig. 6 are the frequency dependences of amplitudes of a couple of surface *TM*-harmonics propagating along the *Oy*-axis, where $|b2(q,s)|$ stands for the amplitude of the surface *TM*-wave. As can be seen, a strong localization of the electromagnetic field at the trapped-mode

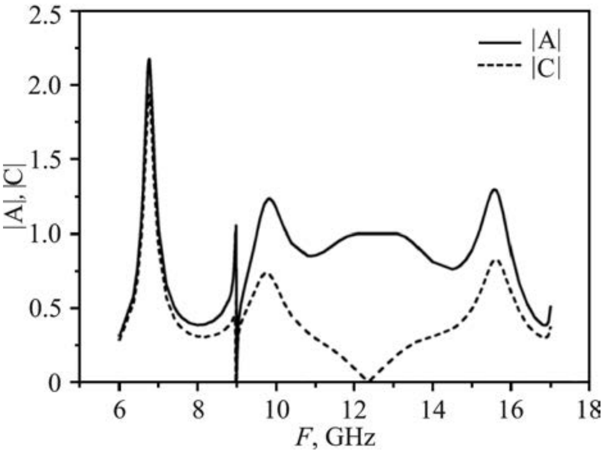

Fig. 4. Magnitudes of the spatial harmonics propagating in the metaresonator volume depending on frequency

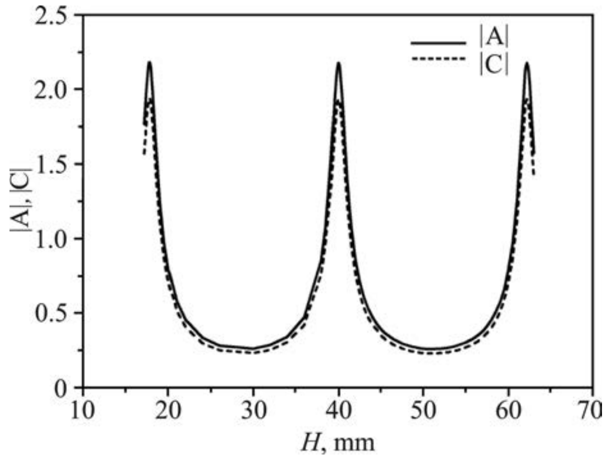

Fig. 5. Electromagnetic field magnitude in the resonator volume at the resonance frequency independence on mirror spacing, $f = 6.76$ GHz

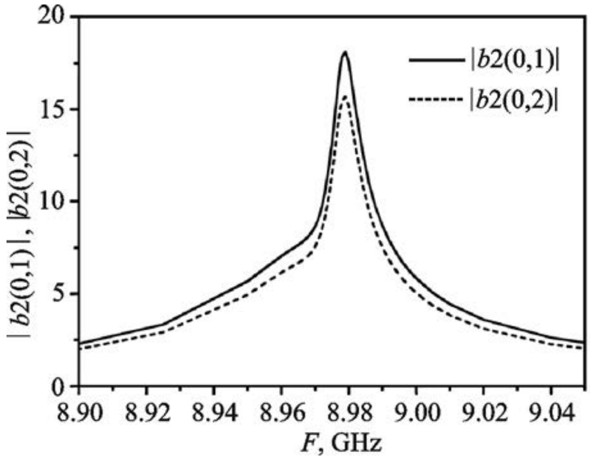

Fig. 6. Frequency dependences of amplitudes of a couple of the surface *TM*-harmonics propagating along the *Oy*-axis

resonance frequency $f = 8.969$ GHz is observed near the surfaces of the metaresonator mirrors. The surface wave amplitude is over tenfold greater than the exciting field amplitude.

The third low-quality resonance of the total transmission of the electromagnetic wave through the metaresonator is observed at the frequency of $f = 9.86$ GHz ($\lambda = 30.43$ mm), which is as well lower than the cutoff frequency of the rectangular waveguides. This resonance is associated with the familiar half-wave resonance in the slots in the metaresonator mirrors. Approximately one half of the free-space wavelength is placed along each slot.

4. Conclusions

Thus, the paper gives the results of numerical investigation of frequency selective properties of a Fabry-Perot resonator formed by metasurfaces, which support the trapped-mode resonance regime. It has been shown that the total transmission of electromagnetic waves through the evanescent holes in the metaresonator mirrors is possible at resonance frequencies. This effect is accompanied by a strong localization of the electromagnetic field both in the resonator volume and near its mirror surfaces. The electromagnetic field amplitude inside the resonator can increase more than twice as compared to the exciting field amplitude. The surface wave amplitude at the resonance frequency shows a dozen increase near the resonator mirrors as against that shown by the incident plane wave. The unique properties of the suggested Fabry-Perot metaresonator can find a wide range of applications in the antenna engineering, as well as in various devices as filters, sensors, etc.

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О. В. Грибовський

Радіоастрономічний інститут НАН України,
вул. Мистецтв, 4, м. Харків, 61002, Україна

МЕТАРЕЗОНАТОР ФАБРІ–ПЕРО, ЯКИЙ ПІДТРИМУЄ РЕЗОНАНСНИЙ РЕЖИМ НА ЗАМКНЕНИХ МОДАХ

Предмет і мета роботи: Дослідження електродинамічних властивостей метарезонатора Фабрі–Перо, утвореного двома паралельними ідеально провідними двовимірно-періодичними двоелементними екранами скінченої товщини з прямокутними отворами. Збудження резонатора здійснюється плоскою лінійно поляризованою електромагнітною хвилею. Базова комірка кожного з екранів, які виконують роль дзеркал метарезонатора, містить два відрізки прямокутних хвилеводів різних поперечних перерізів.

Методи і методологія: Операторний метод розв'язання тривимірних задач дифракції електромагнітних хвиль на багатоелементних двовимірно-періодичних структурах ґрунтується на методі часткових областей і методі узагальнених матриць розсіювання.

Результати: В результаті виконаного чисельного моделювання показано, що модуль коефіцієнта відбиття плоскої хвилі від метарезонатора дорівнює нулю на фіксованих частотах, значення яких нижче частот відсічки для відрізків прямокутних хвилеводів, виконаних в дзеркалах резонатора. На першій, найнижчій, частоті повного проходження електромагнітної хвилі через метарезонатор, спостерігається сильна локалізація електромагнітного поля в його об'ємі. Це пов'язано зі збудженням метарезонатора експоненціально згасаючим полем, яке проникає в об'єм резонатора через позамежні отвори на резонансній частоті. Другий низькочастотний резонанс повного проходження електромагнітної хвилі через метарезонатор пов'язаний з резонансом на замкнених модах, який реєструється в багатоелементних двовимірно-періодичних структурах. У цьому випадку спостерігається сильна локалізація електромагнітного поля поблизу поверхні дзеркал метарезонатора з обох сторін.

Висновки: Унікальні електродинамічні властивості метарезонатора можуть знайти застосування в пристроях для вимірювання електрофізичних параметрів композиційних матеріалів з великими втратами. Властивість сильної локалізації електромагнітного поля як в об'ємі резонатора, так і поблизу поверхонь його дзеркал може бути використана для контролю газоподібних речовин в місцях великого скупчення людей.

Ключові слова: двовимірно-періодичний екран, прямокутний хвилевід, метарезонатор Фабрі–Перо, коефіцієнт відбиття, позамежний хвилевід, резонанс на замкнених модах

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