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CONTROL OVER HIGHER-ORDER TRANSVERSE MODES IN A WAVEGUIDE-BASED QUASI-OPTICAL RESONATOR

Subject and Purpose. *The problems under consideration concern selection and focusing of higher-order modes in a waveguide-based dielectric laser. The purpose is to clarify the physics underlying the behavior of, and permitting control over, continuous terahertz-frequency laser beams of various spatial polarizations.*

Methods and Methodology. *The mode parameters of the waveguide-based laser resonator involving an inhomogeneous phase-stepped mirror were calculated in a matrix technique. To analyze the propagation and focusing of the laser beams that can be excited in a variety of diffraction zones by the wave modes of a waveguide-based quasi-optical resonator, a vectorial Rayleigh—Sommerfeld theory was used. The pertinent experimental studies were performed with the use of known measurement methods suitable for the terahertz frequency range.*

Results. *A method for selecting the higher-order* EH_{12q}*-mode of a terahertz-range laser resonator has been suggested, substantiated theoretically and approbated in experiment. It envisages placing an additional element to perform control over the system's modal structure, namely a (2.3…2.8)* λ*-wide groove on the surface of one of the resonator mirrors. This measure can significantly increase losses for all undesirable modes. At the same time, the losses for the higher* EH_{12q}*-mode remain practically unchanged, which creates conditions for its predominant excitation. Theoretical and experimental studies of moderate and 'sharp' focusing in free space of higher-order modes with different spatial polarizations of a dielectric waveguide-based resonator have been carried out.*

Conclusion. *As has been shown, the proposed phase-stepped mirror with a groove can effectively select the higher-order transverse modes that may be required. The linearly polarized* EH_{12q}*-mode has maximum field intensity in the focal region of the lens employed. For azimuthally polarized* TE_{02q}*- and* TE_{03q}*-modes the central lobes, noticeably shifted from the focus of the lens, have a field maximum. An increase in the axial intensity is observed upon 'sharp' focusing in the field distribution of the radially polarized* TM_{02q}*- and* TM_{03q}*-modes. In this case their central lobes, like those of the higher* TE_{0nq}*-modes, are noticeably shifted from the lens focus.*

Keywords: *terahertz laser, dielectric resonator, inhomogeneous mirror, polarization, selection, focusing, high-order modes.*

Introduction

Terahertz (THz) laser beams can be used for diagnosing the condition of thin films or material surfaces, studying biological objects, and achieving subwavelength resolution in tomography. Other application areas include information transfer, data and image

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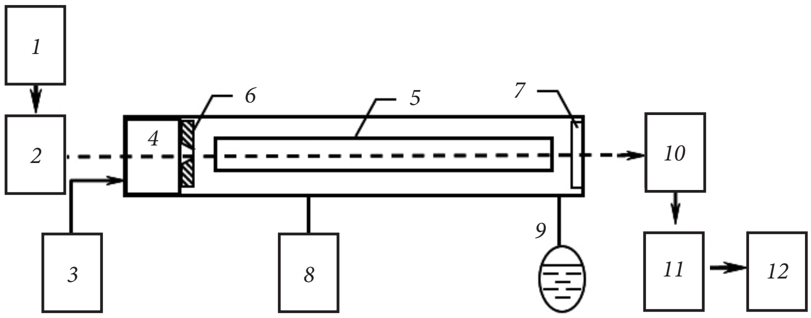

Fig. 1. Block diagram of the experimental setup: *1* — CO_2 laser; *2* — system of folding mirrors; *3* — electric drive; *4* — mirror motion unit; *5* — hollow dielectric waveguide; *6* — input mirror; *7* — output mirror; *8* — THz cell evacuation system; *9* — HCOOH-containing retort; *10* — pyroelectric sensor; *11* — selective amplifier; *12* — ADC and computer

processing, communication systems and lithography [1]. Lately, much attention has been paid to the possibilities provided by laser radiation of radial and azimuthal polarization. As has been shown in a number of works [2, 3], such radiation has a significant potential for production of optomagnetic devices, making it possible to improve focal spot configuration, reduce focal length, and achieve a greater focusing depth.

The studies on the selection and focusing of higher order beams are currently of great interest. Most of the papers published in the area concern the optical range [4–6]. The authors [7] proposed a method based on Poincaré sphere representations of polarization states of radially polarized high-order Laguerre—Gauss vectorial beams. 'Sharp' focusing of higher-order beams of different spatial polarizations was studied in [8]. Paper [9] was a discussion of 'sharp' focusing of polarization-inhomogeneous, higher-order laser beams as effectuated by near-field microscopy methods.

In the THz-range, quite a number of works are known dedicated to problems of selecting and focusing laser beams of different spatial polarizations. Thus, in paper [10] a method of spatial filtering is described that ensures selective excitation of higher-order transverse modes in a waveguide-based quasi-optical resonator. The method is characterized by a high degree of discrimination of undesirable oscillation modes. It is based on the use of a mirror capable of absorbing or scattering the radiation inhomogeneities located at discrete points on its surface. An effective excitation of such modes in the THz-range (specifically, at the wavelength $\lambda = 0.4326$ mm) in a waveguide-based dielectric resonator with an inhomogeneous amplitude-stepped mirror has been confirmed both theoretically and experimentally. In paper [11], a method of selecting the higher-order waveguide mode was proposed, in which the first (along the radial) point of zero field strength coincides with the edge of the output orifice, such that the edge diffraction effect is suppressed. A method for obtaining non-Gaussian light beams with azimuthal field polarization in a waveguide-based quasi-optical cavity is described in paper [12]. The method is based on the use of polarization-selective diffraction structures, such as laser mirrors. The physical features of lower-order modes with different field polarizations were established in paper [13]. The modes considered were excited in a laser resonator based on a THz-band circular dielectric waveguide in the course of either moderate or 'sharp' focusing in a free space. However, the current acute problem, in what concerns such generators, is the selection of transverse modes of different orders, capable of providing single-mode generation, and study of the focusing process.

The purpose of this work is to establish the physical grounds for selecting and focusing higher-order modes of a dielectric waveguide-based resonator for a THz-range laser.

1. Selecting a higher-order mode of linear polarization

The block diagram of an optically pumped, waveguide-based THz laser, and the experimental setup for its study are presented in Fig. 1. Both were described in detail in paper [12]. The working molecule of the HCOOH laser is excited by a dc-discharge preionized CW CO_2 laser as described in paper [14]. The terahertz cell is the vacuum chamber *5* made of Pyrex in the form of a circular tube of internal diameter 35 mm and 1848 mm length, closed by mirrors *6* and *7*. The surface profiles of these mirrors were chosen, based on the need to obtain generation of a desired transverse mode.

The system for detecting the terahertz radiation consists of a pyroelectric detector *10* arranged in a special-purpose electro-mechanical unit and intended for scanning the transverse distribution of the output laser intensity from prescribed azimuths. The detector is placed at a distance from the cell's output mirror *7* ranging from 10 cm to 1.5 m. The spatial resolution of the detector can be adjusted by means of diaphragms placed at its input. The volt-watt sensitivity and the expected width of the radiation beam were taken into account when selecting the resolution of the optical receiver. When measuring the spatial distribution of radiation intensity, the diameter of the diaphragm was chosen in the range from 3.0 mm to 0.3 mm.

The polarization state of the generated mode was determined as follows. The radiation receiver with a small input aperture was moved along different azimuthal directions in the transverse plane of the radiation beam, and the polarization plane's position was determined, with the use of a polarizer, at the points of maximum radiation levels. The polarizer was represented by a one-dimensional wire grating with a 40 μm step and wire diameter of 8 μm.

The power radiated from the THz laser was estimated with a bolometric power transducer BIMO-1.

The output mirror *7* of the cavity (Fig. 1) was a capacitive 2D grid fabricated by depositing aluminum through a matrix onto a 4-mm thick parallel-plane plate of crystalline quartz. The input reflector *6* was a non-uniform mirror involving a central coupling hole of diameter d = 3 mm. The calculations performed showed that with a coupling hole of this diameter, field magnitudes over the mirror are practically not different from such on a mirror without an hole.

Using the matrix technique of paper [15], we have been able to suggest a suitable surface profile of the input mirror *6* for efficient selection of the EH_{12q}-mode in the waveguide-based quasi-optical resonator. In order to discriminate between the transverse modes on the surface of the mirror, it was proposed to make a groove of a $\lambda/4$ depth. In that case, the modulus of the reflection coefficient from the mirror is the same over the entire surface, while its phase in the groove is shifted by π relative to the rest of the mirror surface. As a result, a wave incident on the groove is reflected in antiphase and interferes negatively with the wave reflected from the main part of the mirror. This can greatly reduce the quality factor of all the modes for which the groove would happen to be located close to the area of maximum field amplitude. The calculated dependences of power loss of the resonator modes per round trip over the groove width b of the diffractive phase-stepped mirror are shown in Fig. 2. It can be seen from the Figure that, as a result of placing the groove on a mirror of width b = 1.0...1.2 mm (2.3…2.8) λ, the losses for all undesirable modes can be made to increase as much as possible, while the losses for the highest EH_{12q}-mode would remain practically unchanged. Taking into account the calculation results, the reflective surface of this mirror has been proposed to be made with a groove of width b = 1.1 mm (Fig. 3).

The spectra of the laser modes excited that were obtained experimentally, with the use of the homogeneous input plane and the inhomogeneous phase-stepped mirrors proposed, are shown in Fig. 4. A uni-

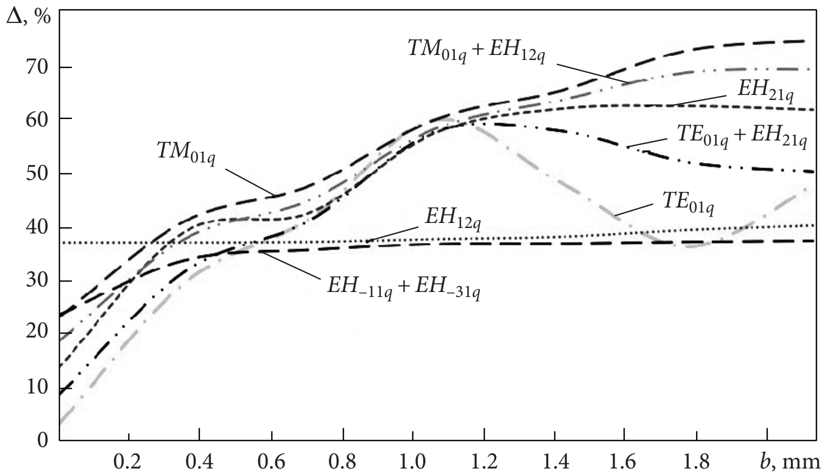

Fig. 2. Calculated dependences upon groove width b of per-round-trip energy losses Δ for modes of the resonator under study (the groove is on the surface of the diffractive mirror)

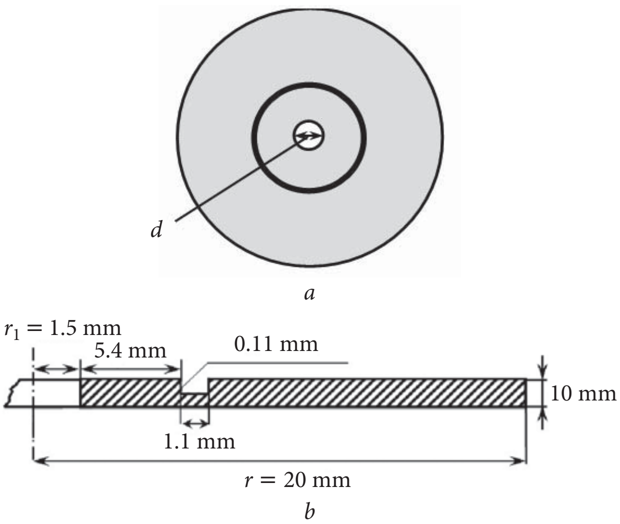

Fig. 3. Profile of the phase-stepped diffractive mirror at the input: surface view (*a*) and transverse cross section of the mirror (*b*)

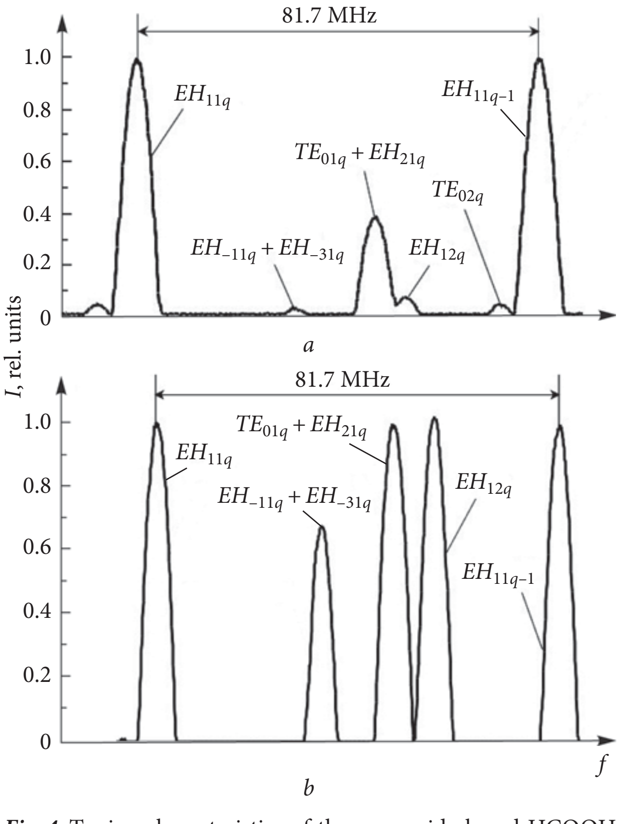

Fig. 4. Tuning characteristics of the waveguide-based HCOOH laser: with a plane mirror at the input (*a*) and with a phase-stepped mirror at the input (*b*)

form two-dimensional capacitive grid served as an output mirror. The resonator length was used as a variable parameter for tuning the process of mode excitation (see Figs. 4, *a* and *b*). In the case of using the phase-stepped mirror, four resonator modes could be observed in the resonator spectrum, which were of linear polarization and showed almost equal radiation powers (Fig. 4, *b*). The radiation power of the THz laser present in the EH_{11q}-, TE_{01q}+EH_{21q}-, and EH_{-11q}+EH_{31q}-modes was 8.0 mW, while in the EH_{12q}-mode it equaled 7.9 mW. These modes were identified, based on the transverse field distributions and intermodal intervals observed [16].

For a resonator with a no-groove reflector *6*, the transverse distribution of the radiation intensity corresponds to such for the fundamental EH_{11}-mode. When the phase-stepped mirror is used, the intensity distribution obtained experimentally shows a qualitative agreement with the one calculated for the EH_{12}-mode. The transverse distribution of the intensity radiated as the EH_{12q}-mode (i.e., the far field at the laser output) is shown in Fig. 5.

So, the proposed phase-stepped mirror having a groove on its surface can effectively select the required higher-order transverse mode.

2. Higher-order mode focusing with different spatial polarization

2.1. Theoretical relations

The modes of the laser resonator that has been used herein for focusing THz-frequency radiation coincide with the modes of a circular hollow dielectric waveguide [17]. Within the initial plane at the output mirror, we can present the radiation in the form of higher-order modes TE_{0nq}-, TM_{0nq}-, and EH_{1nq}-modes ($n > 1$) of a circular dielectric waveguide of radius a_1, that are polarized, respectively, azimuthally, radially, and linearly (Fig. 6). The cylindrical-frame components of these modes, taken within the plane of field source, are described by the expressions [17],

TE_{0nq}-mode

$$\begin{cases} \vec{E}_r(r,\phi) = 0, \\ \vec{E}_\phi(r,\phi) = A_{0n}\dfrac{\chi_{0n}}{a_1}J_1\left(\chi_{0n}\dfrac{r}{a_1}\right), \end{cases} \quad (1)$$

TM_{0nq}-mode

$$\begin{cases} \vec{E}_r(r,\phi) = B_{0n}\dfrac{1}{a_1}J_1\left(\chi_{0n}\dfrac{r}{a_1}\right), \\ \vec{E}_\phi(r,\phi) = 0, \end{cases} \quad (2)$$

EH_{1nq}-mode

$$\begin{cases} \vec{E}_r(r,\phi) = C_{1n} J_0\left(\chi_{1n} \dfrac{r}{a_1}\right) \sin(\phi), \\ \vec{E}_\phi(r,\phi) = C_{1n} J_0\left(\chi_{1n} \dfrac{r}{a_1}\right) \cos(\phi), \end{cases} \quad (3)$$

where $A_{0n} = \dfrac{1}{\sqrt{\pi}\chi_{0n} J_0(\chi_{0n})}$, $B_{0n} = \dfrac{1}{\sqrt{\pi} J_2(\chi_{1n})}$, and $C_{11} = \dfrac{1}{a\sqrt{\pi} J_1(\chi_{1n})}$ are normalizing factors; J_0, J_1 and J_2 are Bessel functions of the first kind, and χ_{0n} and χ_{1n} are roots of the equations $J_0(\chi_{0n}) = 0$ and $J_1(\chi_{1n}) = 0$.

Making use of the Rayleigh—Sommerfeld vectorial theory [18] (the non-paraxial approximation thereof), and recalling Eqs. (1)—(3), we can find the field components of the TE_{0nq}-; TM_{0nq}-, and EH_{1nq}-modes of the dielectric resonator at the input aperture of the lens. Taking into account the phase correction function of the lens [19] and applying once again the Rayleigh—Sommerfeld integrals, we write down the field components of these modes in the focal region area of the lens.

The field components for azimuthally polarized TE_{0nq}-modes at a distance z_2 in the focal region of the lens (Fig. 6) are as follows,

$$\begin{cases} E_r(\rho_2,\theta_2,z_2) = 0, \\ E_\phi(\rho_2,\theta_2,z_2) = -\dfrac{k^2 z_1 z_2}{\xi_2^{\,2}} \exp(ik\xi_2) \times \\ \times A_{0n} \displaystyle\int_0^{a_2} \dfrac{\exp(ik\xi_1)}{\xi_1^{\,2}} \int_0^{a_1} J_1\left(\chi_{0n}\dfrac{r}{a_1}\right) \times \\ \times J_1(\gamma_1 r) \exp\left(\dfrac{ikr^2}{2\xi_1}\right) r dr J_1(\gamma_2\rho_1) \times \\ \times \exp\left(\dfrac{ik\rho_1^{\,2}}{2\xi_2}\right) Ph(\rho_1)\rho_1 d\rho_1, \\ E_z(\rho_2,\theta_2,z_2) = 0, \end{cases} \quad (4)$$

where $k = 2\pi/\lambda$ is the wavenumber; λ is the wavelength; z_1 is the distance from the resonator's output mirror to the lens; z_2 is the distance from the focal plane of the lens to the receiver; ρ_1, θ_1, z_1 stand for cylindrical coordinates within the lens plane $z = z_1$; $\xi_1 = \sqrt{z_1^2 + \rho_1^2}$; $\gamma_1 = k\rho_1/\xi_1$; ρ_2, θ_2, z_2 are the cylindrical coordinates of points within the observation plane $z = z_2$ behind the lens; $\xi_2 = \sqrt{(F + z_2)^2 + \rho_2^2}$; $\gamma_2 = k\rho_2/\xi_2$; a_1 is the radius of waveguide and a_2 the radius of the lens; a_3 is that of the absorbing mask; $Ph(\rho_1) = \exp\left(-i\pi\rho_1^2/\lambda F\right)$ is the phase correcting function for the lens, and F is the focal length.

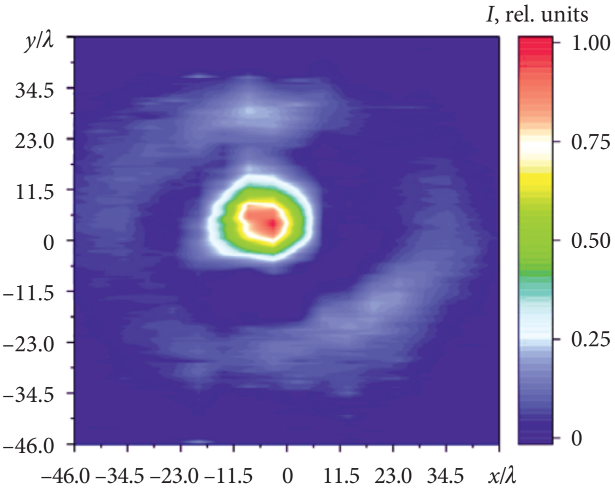

Fig. 5. Measured transverse distribution of the radiation intensity I of the EH_{12q}-mode at a distance of 300 mm from the output mirror

The field components belonging to radially polarized TM_{0nq}-modes are (in the focal region of the lens, at a distance z_2):

$$\begin{cases} E_r(\rho_2,\theta_2,z_2) = \dfrac{k^2 z_1 z_2}{\xi_2^2} \exp(ik\xi_2) \times \\ \times B_{0n} \displaystyle\int_0^{a_2} \dfrac{\exp(ik\xi_1)}{\xi_1^2} \int_0^{a_1} J_1\left(\chi_{0n}\dfrac{r}{a_1}\right) J_1(\gamma_1 r) \times \\ \times \exp\left(\dfrac{ikr^2}{2\xi_1}\right) r dr J_1(\gamma_2\rho_1) \exp\left(\dfrac{ik\rho_1^2}{2\xi_2}\right) Ph(\rho_1)\rho_1 d\rho_1, \\ E_\phi(\rho_2,\theta_2,z_2) = 0, \\ E_z(\rho_2,\theta_2,z_2) = \dfrac{ik^2 z_1}{\xi_2^2} \exp(ik\xi_2) \times \\ \times B_{0n} \displaystyle\int_0^{a_2} \dfrac{\exp(ik\xi_1)}{\xi_1^2} \int_0^{a_1} J_1\left(\chi_{0n}\dfrac{r}{a_1}\right) J_1(\gamma_1 r) \times \\ \times \exp\left(\dfrac{ikr^2}{2\xi_1}\right) r dr \left[\rho_1 J_0(\gamma_2\rho_1) + i\rho_2 J_1(\gamma_2\rho_1)\right] \times \\ \times \exp\left(\dfrac{ik\rho_1^2}{2\xi_2}\right) Ph(\rho_1)\rho_1 d\rho_1. \end{cases} \quad (5)$$

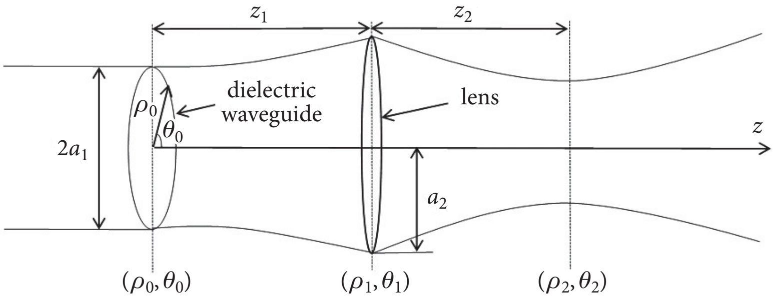

Fig. 6. Computational model of laser beam focusing

The field components for linearly polarized EH_{1nq}-modes in the focal region of the lens, at a distance z_2, are of form

$$\begin{cases} E_r(\rho_2,\theta_2,z_2)=\dfrac{k^2 z_1 z_2}{\xi_2^2}\exp(ik\xi_2)\sin(\theta_2)\times \\ \times C_{1n}\displaystyle\int_0^{a_2}\frac{\exp(ik\xi_1)}{\xi_1^2}\int_0^{a_1}J_0\left(\chi_{0n}\frac{r}{a_1}\right)\times \\ \times J_0(\gamma_1 r)\exp\left(\dfrac{ikr^2}{2\xi_1}\right)rdrJ_0(\gamma_2\rho_1)\times \\ \times\exp\left(\dfrac{ik\rho_1^2}{2\xi_2}\right)Ph(\rho_1)\rho_1 d\rho_1, \\ E_\phi(\rho_2,\theta_2,z_2)=\dfrac{k^2 z_1 z_2}{\xi_2^2}\exp(ik\xi_2)\cos(\theta_2)\times \\ \times C_{1n}\displaystyle\int_0^{a_2}\frac{\exp(ik\xi_1)}{\xi_1^2}\int_0^{a_1}J_0\left(\chi_{0n}\frac{r}{a_1}\right)\times \\ \times J_0(\gamma_1 r)\exp\left(\dfrac{ikr^2}{2\xi_1}\right)rdrJ_0(\gamma_2\rho_1)\times \\ \times\exp\left(\dfrac{ik\rho_1^2}{2\xi_2}\right)Ph(\rho_1)\rho_1 d\rho_1, \\ E_z(\rho_2,\theta_2,z_2)=\dfrac{ik^2 z_1}{\xi_2^2}\exp(ik\xi_2)\sin(\theta_2)\times \\ \times C_{1n}\displaystyle\int_0^{a_2}\frac{\exp(ik\xi_1)}{\xi_1^2}\int_0^{a_1}J_0\left(\chi_{0n}\frac{r}{a_1}\right)\times \\ \times J_0(\gamma_1 r)\exp\left(\dfrac{ikr^2}{2\xi_1}\right)rdr\times \\ \times\left[i\rho_1 J_1(\gamma_2\rho_1)+\rho_2 J_1(\gamma_2\rho_1)\right]\times \\ \times\exp\left(\dfrac{ik\rho_1^2}{2\xi_2}\right)Ph(\rho_1)\rho_1 d\rho_1. \end{cases} \quad (6)$$

2.2. Calculated results and analysis

Eqs. (4)—(6) have been used to calculate the total field intensity of the TE_{0nq}, TM_{0nq}-, and EH_{1nq}-modes, as well as of the longitudinal component of the TM_{0nq}-modes, all in a close vicinity of the minimum-sized focus spot of the beam, for the cases of both 'sharp' and moderate focusing [13]. The wavelength of the radiation under study was 0.4326 mm (emission line of an optically pumped HCOOH THz laser). The waveguide diameter was chosen as $2a_1 = 35$ mm, and the lens diameter equaled $2a_2 = 50$ mm. The focal length F of the lens was chosen according to the conditions of either 'sharp' (the numerical aperture of the lens [20] equaled $NA = 0.68$, where $NA = a_2/F$), or moderate focusing ($NA = 0.16$). The distance z_1 was chosen to be 300 mm, to ensure complete beam interception. The calculations were made for a fixed value of the angle $\theta_2 = \pi/2$.

The regions of maximum field intensity located at a distance $z_{I\max}$ from the geometric focus of the lenses were identified, and the diameters d_σ of the radiation beams in those regions calculated, both for moderate and 'sharp' focusing. Due to the complex structure of the higher-order mode field, their diameter was calculated using the formula [21],

$$d_\sigma = 2\sqrt{\frac{2\int_0^{2\pi}\int_0^{\infty}\rho_2{}^2 I(\rho_2,\theta_2,z_2)\rho_2 d\rho_2 d\theta_2}{\int_0^{2\pi}\int_0^{\infty}I(\rho_2,\theta_2,z_2)\rho_2 d\rho_2 d\theta_2}}. \quad (7)$$

The total field intensity distributions for the linearly polarized EH_{12q}- and EH_{13q}- modes are shown in Fig. 7, for both moderate and 'sharp' focusing. The total intensity of the field of these modes, determined by all three components, has a maximum on the axis.

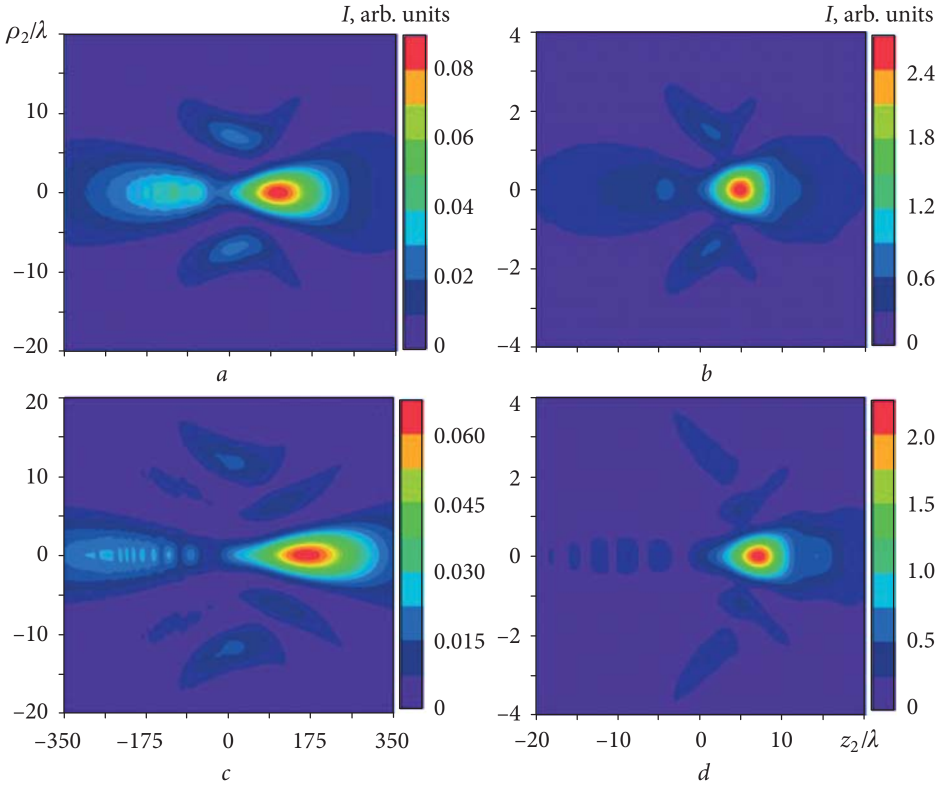

Fig. 7. Calculated distributions of the total field intensity of the EH_{12q} (*a*, *b*) and EH_{13q} (*c*, *d*) modes for moderate (*a*, *c*) and 'sharp' (*b*, *d*) focusing in the focal region of the lens

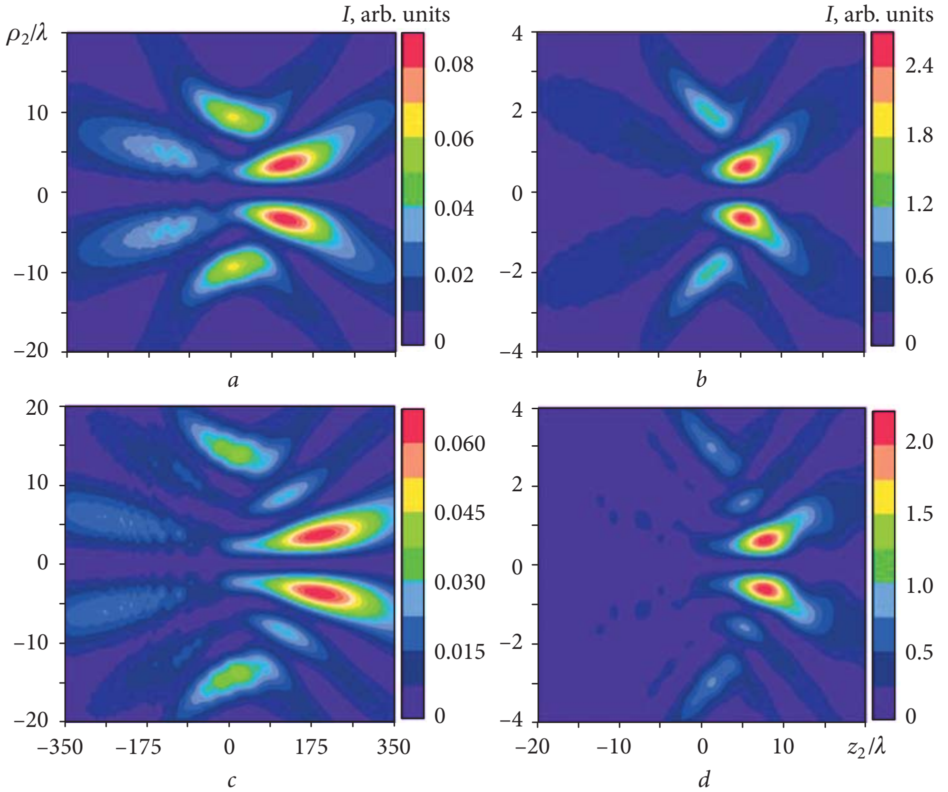

Fig. 8. Calculated distributions of the total field intensity of the TE_{02} (*a*, *b*) and TE_{03} (*c*, *d*) modes for moderate (*a*, *c*) and 'sharp' (*b*, *d*) focusing in the focal region of the lens

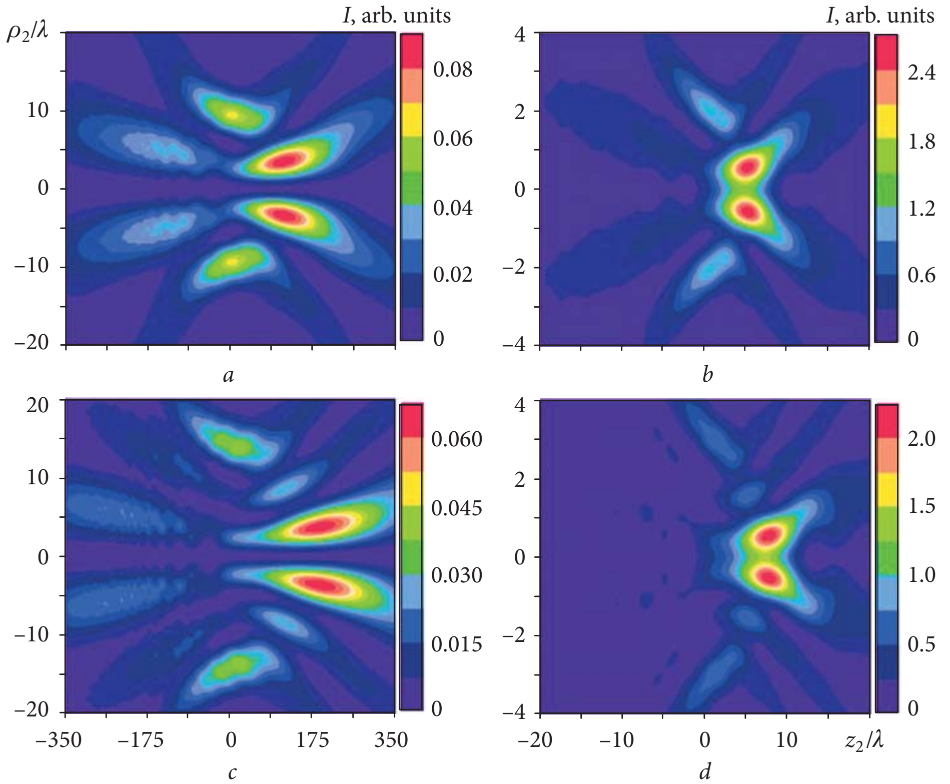

Fig. 9. Calculated distributions of the total field intensity of the TM_{02} (*a*, *b*) and TM_{03q} (*c*, *d*) modes for moderate (*a*, *c*) and 'sharp' (*b*, *d*) focusing in the focal region of the lens

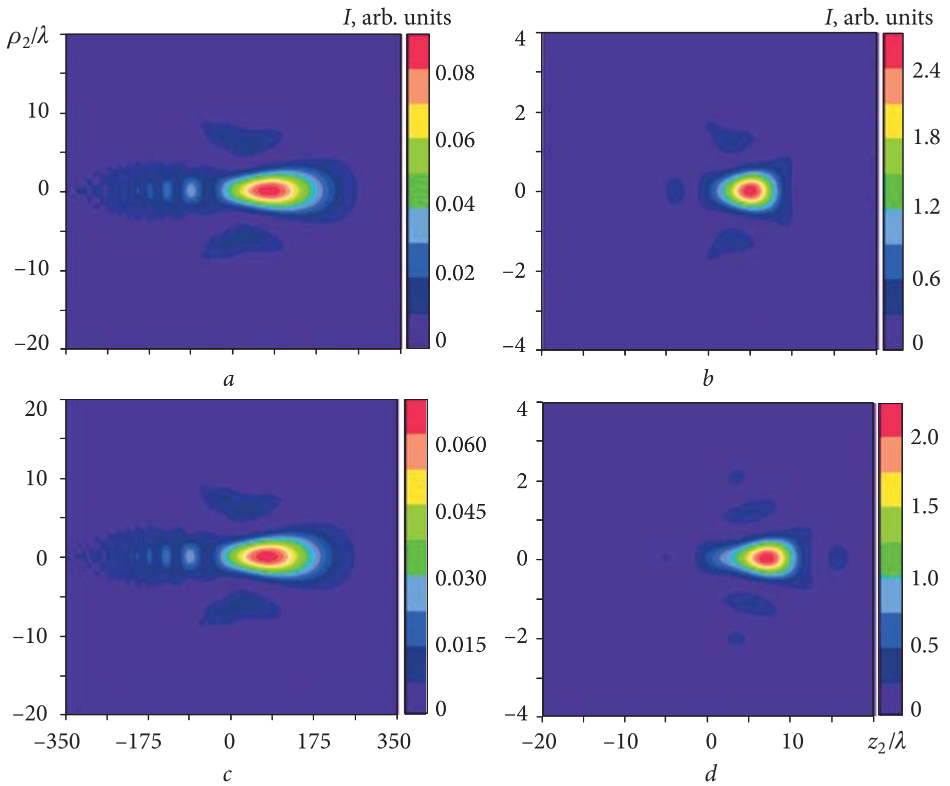

Fig. 10. Calculated intensity distributions of the longitudinal field component of the TM_{02q} (*a*, *b*) and TM_{03q} (*c*, *d*) modes for moderate (*a*, *c*) and 'sharp' (*b*, *d*) focusing in the focal region of the lens

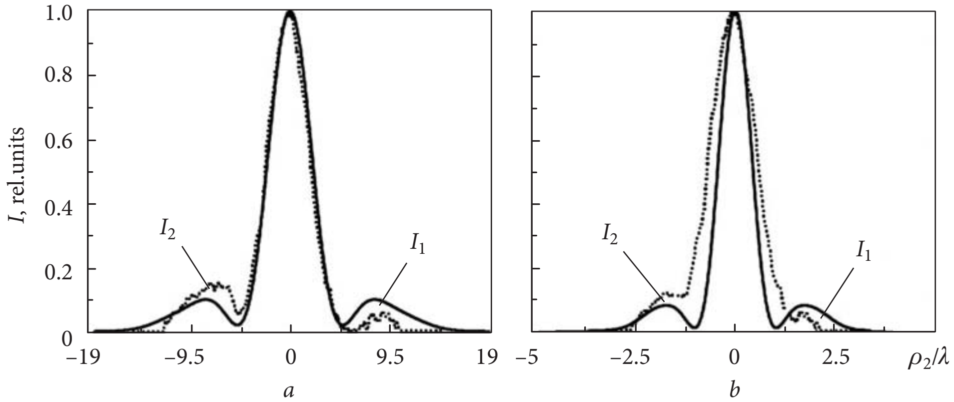

Fig. 11. Calculated (I_1) and measured (I_2) transverse distributions of the total field intensity of the EH_{12q}-mode for cases of moderate (*a*) and 'sharp' (*b*) focusing

The total field distributions of the azimuthally polarized TE_{02q}- and TE_{03q}- modes are presented, for the cases of 'sharp' and moderate focusing, in Fig. 8. The transverse distribution of the total intensity in the near-focal area, characteristic of the azimuthally polarized TE_{02q}- and TE_{03q}- modes of a dielectric resonator, retains its annular form in the cases of both moderate and 'sharp' focusing. The central lobes of these modes, noticeably shifted from the focus of the lens, possess a field maximum. In addition, the total intensity for both modes is determined by a single transverse component described by Eq. (4).

The total field intensity distributions for the radially polarized TM_{02q}- and TM_{03q}- modes (both moderately and strongly ('sharply') focused) are shown in Fig. 9. It can be seen from the graphs that in the case of 'sharp' focusing, the transverse distribution of the field of these modes shows kind of an increase in axial intensity (Fig. 9, *b*, *d*), which is not revealed in the case of moderate focusing (Fig. 9, *a*, *c*). The longitudinal field component of 'sharply' focused TM_{02q}- and TM_{03q}- modes makes a significant contribution into the total intensity of these modes (Fig. 10).

The calculated results concerning location of maximum intensity areas for the higher-order modes (and their separation $z_{I\max}$ from the focus), as well as the effective diameter of such an area, are given in Table. The linearly polarized EH_{12q}-mode displays a minimum value of the beam diameter in close proximity to an elevated-intensity area, both for the 'sharp' and moderate focusing. The maximum field intensity of the azimuthally polarized TE_{03q}-mode is located, for both types of focusing, at maximum distances $z_{I\max}$ from the geometric foci of the lenses under study.

2.3. Comparison of experimental and numerical results

The output homogeneous and input inhomogeneous phase-stepped mirror (Fig. 3) were used to study the focusing of higher-order modes in the THz laser cavity. The laser was tuned to support and amplify the linearly polarized EH_{12q}-mode (Figs. 4 and 5). The free-space diameter of the laser beam in this mode, as estimated in the experiment, was $\mathrm{FWHM}_{\mathrm{exp}} = 16.5\,\lambda$, while the calculated value equaled $\mathrm{FWHM}_{\mathrm{calc}} = 15.96\,\lambda$.

A long-focus lens with a focal length of 160 mm and a short-focus lens with a focal length of 36.36 mm were placed to completely intercept the beam at a distance of $z = 300$ mm. The transverse size of the radiation beam was measured by moving the receiver *10* (Fig. 1) along the optical axis in the focal regions of the two lenses.

As found in the experiments, the maximum field intensity of the EH_{12q}-mode observable in the mo-

Calculated results of finding the maximum intensity positions of higher-order modes and the diameters of their focal spots

Mode types	$NA = 0.68$		$NA = 0.16$	
	$z_{I\max}/\lambda$	d_σ/λ	$z_{I\max}/\lambda$	d_σ/λ
EH_{12q}	4.71	6.22	75.13	26.50
EH_{13q}	6.80	10.34	139.85	42.99
TE_{02q}	4.95	8.14	98.24	34.10
TE_{03q}	7.95	13.11	164.12	51.95
TM_{02q}	5.18	10.55	90.15	34.48
TM_{03q}	7.26	17.72	153.72	53.17

derate focusing regime could be fixed near $z_{I\max} = 74.80\,\lambda$, while the calculated location was $z_{I\max} = 75.13\,\lambda$. In the case of 'sharp' focusing the experiment the maximum field intensity revealed by that mode was measured at $z_{I\max} = 4.6\,\lambda$, while the calculation predicted $z_{I\max} = 4.71\,\lambda$. The transverse intensity distributions for the EH_{12q}-mode, obtainable at such distances are shown in Fig. 11, both for moderate and 'sharp' focusing. The measured diameter of a moderately focused EH_{12q}-mode ($NA = 0.16$) was $\mathrm{FWHM_{exp}} = 4.50\,\lambda$ (compare with the calculated value $\mathrm{FWHM_{calc}} = 4.48\,\lambda$). In the case of 'sharp' focusing ($NA = 0.68$) the respective values happened to be $\mathrm{FWHM_{exp}} = 0.90\,\lambda$ and $\mathrm{FWHM_{calc}} = 1.27\,\lambda$.

Conclusions

A method for selecting the higher-order EH_{12q}-mode of a THz-range laser resonator has been suggested, substantiated theoretically and approbated in experiment. It is based on placing a groove, $(2.3\ldots2.8)\,\lambda$ in width, on the surface of one of the mirrors in the waveguide-based quasi-optical resonator. This makes it possible to significantly increase the losses for all undesirable modes, while leaving the losses for the highest EH_{12q}-mode practically unchanged, which creates conditions for its predominant excitation.

Theoretical and experimental studies of moderate and 'sharp' focusing of higher-order modes of a dielectric waveguide resonator have been carried out. It is shown that the linearly polarized EH_{12q}-mode is characterized by the maximum field intensity. The central lobes, noticeably shifted from the focus of the lens, have a field maximum for the azimuthally polarized TE_{02q}- and TE_{03q}-modes. In the case of 'sharp' focusing an increase in axial intensity is observed in the field distributions of the radially polarized modes TM_{02q} and TM_{03q} . At the same time, their central lobes, like those of the higher-order modes TE_{0nq}, demonstrate a noticeable shift from the lens focus.

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КЕРУВАННЯ ВИЩИМИ ПОПЕРЕЧНИМИ МОДАМИ ХВИЛЕВІДНОГО КВАЗІОПТИЧНОГО РЕЗОНАТОРА

Предмет і мета роботи. Розглядаються задачі селекції та фокусування мод вищого порядку діелектричного хвилевідного лазера. Мета роботи полягає у встановленні фізичних закономірностей поведінки та можливостей керування лазерними пучками безперервного випромінювання з різною просторовою поляризацією в терагерцовому діапазоні частот.

Методи і методологія. Для розрахунку модових характеристик хвилевідного лазерного резонатора з неоднорідним фазоступінчастим дзеркалом у роботі використовувався матричний метод. Для вивчення поширення та фокусування лазерних пучків, що збуджуються модами хвилевідного квазіоптичного резонатора в різних зонах дифракції, застосовано векторну теорію Релея—Зоммерфельда. У відповідних експериментальних дослідженнях застосовувалися відомі методи вимірювань терагерцового діапазону.

Результати. Запропоновано і теоретично обґрунтовано та експериментально підтверджено метод селекції вищої EH_{12q}-моди в лазерному резонаторі терагерцового діапазону, котрим передбачається розміщення додаткового елемента керування модовим складом — канавки шириною (2.3…2.8) λ на поверхні одного з дзеркал резонатора. Це дозволяє значно збільшити втрати для всіх небажаних мод. При цьому втрати для вищої EH_{12q}-моди залишаються практично незмінними, що створює умови для її переважного збудження. Проведено теоретичні й експериментальні дослідження помірного та гострого фокусування у вільному просторі, за допомогою лінзи, вищих мод діелектричного хвилевідного резонатора, що мають різну просторову поляризацію.

Висновки. Показано, що запропоноване фазоступінчасте дзеркало з канавкою уможливлює ефективну селекцію необхідної вищої поперечної моди. Показано, що лінійно поляризована EH_{12q}-мода має максимальну інтенсивність поля у фокальній області лінзи. Для азимутально поляризованих TE_{02q}- та TE_{03q}-мод максимум поля спостерігається в центральних пелюстках, що є помітно зміщеними від фокуса лінзи. При гострому фокусуванні у радіально поляризованих TM_{02q}- та TM_{03q}-мод у розподілі поля спостерігається зростання осьової інтенсивності. При цьому їхні центральні пелюстки, як і у вищих TE_{0nq}-мод, помітно зміщені від фокуса.

Ключові слова: *терагерцовий лазер, діелектричний резонатор, неоднорідне дзеркало, поляризація, селекція, фокусування, моди вищого порядку.*