### ЕЛЕКТРОМАГНІТНА ТЕОРІЯ Electromagnetic theory

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### A RECONFIGURABLE METASURFACE IN THE FORM OF A FABRY-PÉROT RESONATOR, INVOLVING SEMI-TRANSPARENT MIRRORS AND A NONLINEAR DIELECTRIC INSERT

**Subject and Purpose.** The paper is aimed at suggesting ways for implementing electromagnetic devices that would involve a nonlinear dielectric inside the structure for concentrating electromagnetic field within it at relatively low intensities of the excitation field. A representative example is offered by a Fabry-Pérot resonator with semi-transparent mirrors and a nonlinear dielectric inside.

**Methods and Methodology.** The paper presents a fundamentally new approach to controlling the frequency response of a metasurface, and a method for protecting electronic modules from high excitation powers. The method is based on the use of nonlinear properties of a Fabry-Pérot "resonator-capacitor" containing a nonlinear dielectric in its volume. Thus, we propose a method for creating spatial filters and antenna protecting fairings capable of active reconfiguration.

**Results.** Analysis of the results has shown that by changing the voltage across the resonator-capacitor plates it is possible to control the degree of electromagnetic field localization in the resonator volume. The frequency response of the metasurface based on the Fabry-Pérot resonator with a nonlinear dielectric inside can be reconfigured by changing the voltage applied to the resonator mirrors. The advantages provided by electrical control of the frequency-selective characteristics include an increased efficiency and possibility of integrating digitally controllable systems into antennas.

**Conclusions.** The use of a Fabry-Pérot "resonator-capacitor" model, with a nonlinear dielectric inside, in the capacity of a reconfigurable metasurface, else as a power limiter in a variety of devi-ces, has sufficient prospects for application in microwave transmitting systems. A special area of application for the reconfigurable Fabry-Pérot resonators is creation of broadband receive antenna systems for direction-finding. In essence, the proposed Fabry-Pérot resonator is a reflective antenna array with an ability to exhibit, conceal or alter its electrodynamic properties according to a certain algorithm. This opens up prospects for its application in "friend-or-foe" recognition systems, reflective beacons, navigation systems, etc.

Keywords: Fabry-Pérot resonator, nonlinear dielectric, rectangular holes, reflection coefficient, semitransparent mirrors.

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### Introduction

Currently, studies are underway concerning devices whose properties depend upon intensity of the incident electromagnetic field or the voltage applied. For example, such devices can be implemented on the basis of periodic arrays [1–5] or Fabry-Pérot resonators [6, 7]. For such applications, it is necessary that the arrays or the resonators contained, for example, elements with a nonlinear Kerr-type dielectric [8]. In that case, the permittivity of the nonlinear dielectric depends on the squared absolute value of the electric field inside the dielectric. It should be noted that the essential condition for manifestation of nonlinear properties of the dielectric in a structure is the existence of a sufficiently intense electric field well inside the dielectric.

The fundamental difficulty in practical application of such devices stems from the fact that nonlinear effects in arrays involving a nonlinear dielectric arise at relatively high intensities of the electromagnetic field which is incident on the structure. In a Fabry-Pérot resonator with a nonlinear medium between the mirrors the electromagnetic field intensity at the resonant wavelength inside the resonator can be greatly in excess of the intensity of the external radiation, due to the field localization effect. This phenomenon favors manifestation of nonlinear effects at relatively low intensities of the acting radiation inside the resonator. In addition, the reflection coefficient shown at the resonant wavelength is sensitive to small changes in the refractive index of the medium within the resonant cavity. In this case, the value of the reflection coefficient of such a device is determined both by nonlinear properties of the medium and design parameters of the resonator. Therefore, the aim of this work is to design (and study theoretically) such electrodynamic models with a nonlinear dielectric inside which could concentrate electromagnetic field within the dielectric at relatively low intensities of the exciting field. For example, these could be Fabry-Pérot resonators with semi-transparent mirrors and a nonlinear dielectric inside.

## 1. Investigation of a linear Fabry-Pérot resonator

Consider a structure consisting of two parallel, perfectly conducting screens of infinite extent, characterized by equal thicknesses h and perforated by



**Fig. 1.** The Fabry-Pérot resonator (a), basic cell of the Fabry-Pérot resonator (b)

rectangular openings. The separation between the screens is L, being chosen from the condition  $L > \lambda$ , where  $\lambda$  is the wavelength in free space. Shown in Fig. 1, a is a Fabry-Pérot resonator formed by two parallel screens with rectangular openings. Centers of the base cells of these latter are located periodically at the nodes of a rectangular grid. The openings in the screens are regarded as segments of rectangular waveguides, all possessing identical cross sections  $a \times b$ . The cross section dimensions are selected, for a specific frequency range, from the condition that the waveguide should allow propagation of the lowest-order mode alone. The centers of the basic cells in the two screens are placed with periods  $d_1$  (along the 0x axis) and  $d_2$  (along the 0y axis) (see Fig. 1, b). The periods are selected so as to provide satisfaction of the condition that just one spatial harmonic should exist in free space.

In order to be able creating devices of a new generation, based on nonlinear Fabry-Pérot resonators, it is necessary to understand which intensity levels of the exciting field, as well as of the field inside the resonator allow controlling properties of such devices.



Fig. 2. Cross section of the base cell of the linear resonator



*Fig.* **3**. Frequency dependence of the reflection coefficient's modulus in the linear resonator

It is necessary to obtain an estimate of the field intensity inside, and to understand which nonlinear materials could be best suitable for application in similar devices.

Solution of the problem of plane electromagnetic wave diffraction on nonlinear resonators proceeds from a study of their properties in the linear case, when a wave of a low intensity is incident on a resonator involving a dielectric with a linear electromagnetic response. Details of the algorithm for investigating such structures in the linear regime can be found in [9-11]. The calculation algorithm is based on solution of key problems of electromagnetic wave diffraction by two-dimensional periodic arrays of semi-infinite waveguides characterized by a rectangular cross-section. The generalized scatter matrices for plane electromagnetic waves can be useful in implementation of the operator method for problems of diffraction by multilayered arrays.



*Fig. 4.* Frequency dependences of the electric field amplitudes of the wave propagating through the resonator

Let us place a dielectric plate into the resonator to study the frequency dependent modulus of its reflection coefficient for a plane wave. At this stage of research, we assume that the dielectric permittivity of the plate is independent of the electric field's intensity. Fig. 2 presents a cross-sectional view of the basic Fabry-Pérot resonator cell with a dielectric plate. In the Figure, q denotes the incident wave's amplitude; **B** is the vector of amplitudes of spatial harmonics of the wave reflected from the resonator; A, C,  $A_1$ , and  $C_1$  are amplitudes of spatial harmonics of the electromagnetic wave propagating in the resonator between the mirrors and the dielectric plate, and **D** is the vector of amplitudes of spatial harmonics of the electromagnetic field that has passed through the resonator.

Taking into account the notation of Fig. 2, we can write a set of operator equations for the unknown amplitudes of spatial harmonics of the plane waves/ both inside and outside the resonator, *viz*.

$$\begin{split} \mathbf{B} &= Rq + TC, \\ A &= Tq + RC, \\ C &= e_1 R_{\varepsilon} e_1 A + e_1 T_{\varepsilon} C_1 \\ A_1 &= T_{\varepsilon} e_1 A + R_{\varepsilon} C_1, \\ C_1 &= e_2 R e_2 A_1, \\ \mathbf{D} &= T e_2 A_1, \end{split}$$

where *R*, *T* are the reflection and transmission operators, respectively, for the plane electro-magnetic waves over some screen containing rectangular holes;  $R_{\varepsilon}$ ,  $T_{\varepsilon}$  are the reflection and transmission operators, respectively, for the plane waves within the

dielectric plate, and  $e_{1,2}$  are the operators taking into account the phase shifts obtained by the spatial harmonics along their paths  $h_1$  and  $h_2$  between a screen and the plate.

The solution of the operator equations set is as follows:

$$A = \left(I - Re_1 R_{\varepsilon} e_1 - Re_1 T_{\varepsilon} e_2 Re_2 \times \left(I - R_{\varepsilon} e_2 Re_2\right)^{-1} T_{\varepsilon} e_1\right)^{-1} Tq,$$
  

$$A_1 = \left(I - R_{\varepsilon} e_2 Re_2\right)^{-1} T_{\varepsilon} e_1 A, C_1 = e_2 Re_2 A_1,$$
  

$$C = e_1 R_{\varepsilon} e_1 A + e_1 T_{\varepsilon} C_1, \quad \mathbf{B} = Rq + TC, \quad \mathbf{D} = Te_2 A_1,$$

where *I* is the unit matrix.

The studies were carried out for the case of a plane, linearly polarized electromagnetic wave of unit amplitude q = 1 V/cm incident normally on the resonator. The electric vector in the incident wave is parallel to the axis 0y. The orientation of the polarization vector in the incident wave has been selected from the condition of ensuring an effective excitation of the fundamental wave mode within the rectangular waveguide channels located in the resonator mirrors. Centers of the basic cells of the mirrors forming the Fabry-Pérot resonator are located in the nodes of the square mesh with equal periods  $d_1 = d_2 = d$ . A dielectric plate of thickness  $h_{\varepsilon} = 4 \text{ mm}$  was placed at the center of the resonator  $(h_1 = h_2)$ . The dielectric permittivity of the plate is  $\varepsilon = 4$ . The geometrical parameters of the structure were as follows, d = 17.5 mm; a = 14 mm; b = 1.5 mm, and h = 5 mm. The separation between the mirrors,  $L = h_1 + h_2 + h_3 = 15$  mm, has been chosen from the condition providing for a total transmission, at resonant frequencies, of the electromagnetic wave through the resonator (for given parameters of the dielectric plate).

Figure 3 shows the dependence of the reflection coefficient modulus in the linear resonator with a dielectric plate, at the frequency of the excitation field.

Two cases of total resonant transmission of an electromagnetic wave through the resonator are observable at frequencies F = 16.067 GHz and F = 16.507 GHz. Figure 4 shows frequency dependent electric field amplitudes of an electromagnetic wave which propagates through the resonator in opposite directions. As can be seen, for F = 16.067 GHz there is a small area of electromagnetic field localization in the resonator volume, where the amplitude increases

to become nearly 2.5 times higher than the amplitude of the incident field. In this case, the structure is excited at the resonant wavelength  $\lambda_r = 18.67$  mm. The resonant wavelength pertinent to the structure is slightly larger than the separation between its mirrors. This is due to the fact that the electromagnetic field penetrates into the rectangular slots in the mirrors, so that the effective (resonant) spacing between the resonator mirrors is higher than such in a resonator without holes in its mirrors. Figure 4 shows that at a frequency F = 16.507 GHz of the resonator becomes "transparent" with respect to the incident electromagnetic wave- as the field amplitude A = 1 in the resonator becomes equal to the amplitude of the incident field, whereas the amplitude of the field reflected from the dielectric plate is zero, C = 0.

# 2. Investigation of a nonlinear Fabry-Pérot resonator

We will consider now a resonator containing a dielectric with a nonlinear Kerr-type response. In this case, the magnitude of the dielectric constant of the material is dependent on the squared absolute value of the electric field strength inside the dielectric, i.e.  $|E_{in}|^2$ , thus being directly proportional to the internal field's intensity,  $\varepsilon = \varepsilon_l + \varepsilon_n |I_{in}|$  (here  $\varepsilon$  is the dielectric constant of the nonlinear material;  $\varepsilon_l$  denotes the inear part of the dielectric constant;  $\varepsilon_n$ (cm<sup>2</sup>/kW) is the nonlinearity coefficient, and  $I_{in}$  the electromagnetic field's intensity within the dielectric.

While analyzing electrodynamic properties of two-dimensional periodic lattices with nonlinear response, the writers [1-3] found that the condition necessary for demonstrating nonlinear properties is presence of a strong electromagnetic field inside the dielectric. Therefore, the periodic structures which permit localizing fields of high intensity at resonant frequencies (about  $|I_{in}| = (10-100) \text{ kW/cm}^2$ ) are perfect players when they contain a nonlinear material. The amplitude of the exciting electromagnetic field should be within the range like |A| = (60-150) V/cm. As follows from the above results, the nonlinear structure in the form of a Fabry-Pérot resonator with a dielectric insert operating in the frequency range from 15 to 17 GHz, does not satisfy any of the above requirements because of the weak localization of the electromagnetic field inside the resonator. An order of magnitude or even higher increase in the electro-



Fig. 5. The nonlinear "resonator-capacitor"



*Fig. 6.* Frequency dependent absolute magnitudes of the reflection coefficient, shown for a variety of voltages across the "resonator-capacitor" plates (black lines: U = 0 kV; red lines: U = 50 kV, and blue lines correspond to U = 75 kV)

magnetic field amplitude inside the resonator could be achieved, for example, through reduction in the absolute geometrical dimensions of the resonator with openings adjacent to edges of the mirrors, or else through a transfer toward higher-frequency portions of the operating range [7].

In order to analyze the nonlinear Fabry-Pérot resonator in the selected frequency range, we will consider it as a nonlinear capacitor (varicond), of which the capacitance varies, following some nonlinear law, within a wide range that is, in its turn, dependent on the voltage applied to the plates (see Fig. 5).

Since the dielectric plate is located at the center of the resonator, and the widths of the openings in the resonator mirrors are an order of magnitude smaller than the separation between the mirrors, the effect of the openings upon the electrostatic field strength



*Fig.* 7. Frequency dependences of the electric field amplitudes of the wave propagating through the nonlinear resonator at U = 50 kV

at the resonator center can be neglected. Therefore, the electric field strength inside a dielectric can be approximated to by the formula  $|E_{in}| = E_0 / \varepsilon_l$ , where  $E_0$  is the electric field strength in an empty capacitor, which is approximated by the formula  $E_0 = U/L$ , and  $\varepsilon_l$  the linear part of the dielectric permittivity of the filling material. Here U (kV) is the potential difference across the capacitor plates, and L(cm) is the separation between the plates. So, an approximate expression for the dielectric permittivity of a nonlinear dielectric in this structure is given by the formula  $\varepsilon = \varepsilon_l + \varepsilon_n (U^2 / L^2) / \varepsilon_l^2$ . We will now estimate the potential difference between the plates of the nonlinear "resonator-capacitor" at which the dielectric constant of the nonlinear dielectric would be higher by 1 relative the permittivity at U=0, namely  $\varepsilon_n (U^2/L^2)/\varepsilon_l^2 > 1$ . Taking  $\varepsilon_l = 4$ ,  $\varepsilon_n = 0.005 \text{ cm}^2/\text{kW}$ , and L = 1.5 cm, we arrive at an estimate like U > 84.85 kV.

The potential difference *U* across the "resonator-capacitor" covers can be reduced in several ways. Either the space between the resonator mirrors can be completely filled with a dielectric, or the entire experiment shifted to a higher frequency region (still within the operating range), by simply cutting down the separation between the resonator mirrors.

Figure 6 shows dependences of the reflection coefficient modulus in the nonlinear resonator upon the frequency of the exciting electromagnetic field, for a variety of voltages across the plates.

The graphs show that the frequency response of a Fabry-Pérot resonator-based metasurface involving

a nonlinear dielectric can be tuned by changing the voltage applied to the resonator mirrors. An increase in the potential difference between the "resonatorcapacitor" plates shifts the frequencies of total resonant transmission of the electromagnetic field through the resonator toward lower frequencies, in a contrast to the case of a zero voltage across the plates .This is due to an increase in the dielectric constant of the nonlinear dielectric following the electric field increase in the dielectric. This is in agreement with the familiar effect accompanying operation of open resonators. With an increase in the dielectric constant within the resonator, its resonant frequency shifts towards lower frequencies. In addition, numerical studies have shown that by means of changing the voltage across the "resonator-capacitor" plates it is possible to adjust the degree of electromagnetic field localization in the resonator volume. Figure 7 shows frequency dependences of the field amplitudes of the electromagnetic wave propagating in the resonator, for a U = 50 kV potential difference

between the plates. It can be seen that in this case the field amplitude in the resonator is increased by a factor of 4.5 compared with the amplitude of the primary electromagnetic field at U=0 kV.

### Conclusion

Suggested in the paper has been a fundamentally new method of controlling the frequency response of a metasurface, as well as techniques for protecting electronic modules from excessively high excitation power levels. The method exploits nonlinear properties of a Fabry-Pérot "resonator-capacitor" involving a nonlinear dielectric.

The nonlinear properties of the Fabry-Pérot "resonator-capacitor" model allow treating it as a tunable metasurface, else as a power limiter in a variety of devices, thus opening up prospects for implementation and a set of applications in microwave transceiver systems. A distinctive feature of the model is simplicity of its execution.

#### REFERENCES

- Kochetova, L.A., Prosvirnin, S.L., Tuz, V.R., 2014. Optical bistability in a grating with slits filled nonlinear media. *Prog. Electro*magn. Res., 35, pp. 133–139. DOI:10.2528/PIERM14012606
- 2. Tuz, V.R., Kochetov, B.A., Kochetova, L.A., Mladyonov, P.L., Prosvirnin, S.L., 2015. Two-oscillator model of trapped-modes interaction in a nonlinear bilayer fish-scale metamaterial. *Phys. Scr.*, **90**(2), 025504. DOI: 10.1088/0031-8949/90/2/025504
- Tolmachev, V.A., Melnikov, V.A., Baldycheva, A.V., Berwick, K., Perova T.S., 2012. Electrically Tunable Fabry-Pérot Resonator Based on Microstructured, Si Containing Liquid Crystal. Prog. Electromagn. Res., 122, pp. 293–309.
- Sydorchuk, N.V., Prosvirnin, S.L., Fan, Y., Zhang, F., 2019. Analysis of terahertz wave nonlinear reflection by an array of double silicon elements placed on a metal substrate. J. Phys. D: Appl. Phys., 52, 355303. DOI: 10.48550/arXiv.1903.06074
- Tuz, V.R., Kochetov, B.A., Kochetova, L.A., Mladyonov, P.L., Prosvirnin, S.L., 2015. Two-oscillator model of trapped-modes interaction in a nonlinear bilayer fish-scale metamaterial. *Phys. Scr.*, 90(2), 025504. DOI: 10.1088/0031-8949/90/2/025504
- Flannery, J., Maruf, R. Al, Yoon, T., Bajcsy, M., 2018. Fabry-Pérot cavity formed with dielectric metasurfaces in a hollow-core fiber. ACS Photonics, 5(2), pp. 337–341. DOI: 10.1021/acsphotonics.7b01154
- Fan, K., Koulakis, J., Holczer, K., Putterman, S., Padilla, W.J., 2020. Ultrathin Metasurface Wavelength-Selective Mirror for Millimeter/Terahertz Wave Fabry-Pérot Cavities. J. Infrared Millim Terahertz Waves, 41(1), pp. 365–374. DOI:10.1007/s10762-019-00657-2
- 8. Pezeshki, H., and Ahmadi, V., 2013. All-optical bistable switching based on photonic crystal slab nanocavity using nonlinear Kerr effect. J. Mod. Opt., 60(2), pp. 103–108. DOI: 10.1080/09500340.2012.737033
- 9. Gribovsky, A.V., 2017. A Quasi-Periodic Sequence of the Fabry-Pérot Resonators on the Basis of Planar Screens of Finite Thickness with Rectangular Holes. *Telecommunications and Radio Engineering*, **76**(16), pp. 1417–1422. DOI: 10.1615/TelecomRadEng. v76.i16.30
- Gribovsky, A.V., and Kuzmichev, I.K., 2016. Fabry-Pérot resonator formed by two screens with rectangular holes. *Radio Phys. Radio Astron.*, 21(1), pp. 58–64. DOI: 10.15407/rpra21.01.058
- Gribovsky, A.V., 2019. Sensor for Measuring the Permittivity of Solid and Gaseous Substances on the Basis of a Fabry-Perot Resonator with Evanscent Holes in the Mirrors. *Telecommunications and Radio Engineering*, 78(11), pp. 939–947. DOI: 10.1615/ TelecomRadEng.v78.i11.20

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#### МЕТАПОВЕРХНЯ, ЩО ПЕРЕБУДОВУЄТЬСЯ, У ВИГЛЯДІ РЕЗОНАТОРА ФАБРІ–ПЕРО З НАПІВПРОЗОРИМИ ДЗЕРКАЛАМИ ТА НЕЛІНІЙНИМ ДІЕЛЕКТРИКОМ

**Предмет і мета роботи.** Основною метою роботи є проєктування пристроїв з нелінійним діелектриком, які здатні концентрувати електромагнітне поле всередині діелектрика за порівняно малих інтенсивностей поля збудження, на прикладі резонатора Фабрі-Перо з напівпрозорими дзеркалами та нелінійним діелектриком усередині.

**Методи та методологія.** У роботі представлено принципово новий метод керування частотною характеристикою метаповерхні та спосіб захисту електронних модулів від високих значень збуджувальних потужностей. Метод базується на використанні нелінійних властивостей «резонатора-конденсатора» Фабрі-Перо з нелінійним діелектриком в об'ємі резонатора. Таким чином, пропонується метод створення просторових фільтрів і захисних обтікачів антен, що мають властивість активного переналаштування.

**Результати.** З результатів аналізу випливає, що зміною напрути на обкладках «резонатора-конденсатора» можливо регулювати ступінь локалізації електромагнітного поля в об'ємі резонатора. Частотну характеристику метаповерхні на основі резонатора Фабрі–Перо з нелінійним діелектриком можна переналаштовувати шляхом зміни напруги, що прикладається до дзеркал резонатора. Перевагою електричного керування частотно-селективними характеристиками є швидкодія та можливість інтеграції в антенні системи з цифровим керуванням.

Висновки. Використання моделі «резонатора-конденсатора» Фабрі-Перо з нелінійним діелектриком як метаповерхні, що переналаштовується, або як обмежувача потужності в різних пристроях має достатню перспективу для впровадження й застосування в приймально-передавальних системах НВЧ. Особливим напрямком впровадження резонаторів Фабрі-Перо, здатних до переналаштування, є створення на їхній основі широкосмугових приймальних антенних систем для засобів пеленгації. Власне, запропонований резонатор Фабрі-Перо є відбивною антенною решіткою з можливістю проявляти, приховувати або змінювати за визначеним алгоритмом свої електродинамічні властивості. Це відкриває перспективи використання його в системах розпізнавання свій-чужий, відбивних маяках, системах навігації тощо.

**Ключові слова:** резонатор Фабрі–Перо, нелінійний діелектрик, прямокутні отвори, коефіцієнт відбиття, напівпрозорі дзеркала.