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V.A. Buts[1,2], D.M. Vavriv[2]

[1] National Science Center "Kharkiv Institute of Physics and Technology" of the NAS of Ukraine
1, Akademichna St., Kharkiv, 61108, Ukraine
[2] Institute of Radio Astronomy of the NAS of Ukraine
4, Mystetstv St., Kharkiv, 61002, Ukraine
E-mails: vbuts1225@gmail.com; vavriv@rian.kharkov.ua

PHASE SYNCHRONIZATION OF PARTICLES AT CYCLOTRON RESONANCES

Subject and Purpose. *The effects considered concern phase synchronization of electrons in an ideal plasma subjected to the action of an external uniform, d. c. magnetic field. Two modes of the synchronization are discussed, specifically one by an external electromagnetic field and the other by the cyclotron radiation emitted by the electrons. The purpose is to compare these forms of synchronization and their effects on plasma stability.*

Methods and Methodology. *The plasma is represented as a set of coupled oscillators whose dynamics is described via coupled differential equations. Assuming the coupling between the oscillators to be weak we find analytical solutions to the equation, further performing a stability analysis which exploits standard approaches of the dynamical systems theory. The solutions found are validated through corresponding numerical simulations.*

Results. *As has been found, an external electromagnetic wave may be capable of guiding the particles toward phase synchronization, which can lead to formation of phased bunches. This mechanism of particle grouping may prove to be more efficient, in terms of scale times of synchronization, if compared with known mechanisms exploiting relativistic effects. Additionally, we show that the cyclotron radiation emitted by the charged particles (which is often disregarded because of its smallness) can lead to self-phase synchronization of the electrons. Moreover, should the density of charged particles in the ensemble be sufficiently high, an instability can arise, potentially disrupting the ensemble. Estimates have been provided of the level of random fluctuations capable of undermining the synchronization process and plasma dynamics stabilization.*

Conclusions. *The most significant finding of this analysis is the emergence of low-frequency oscillations in the charged oscillators set, followed by an onset of the plasma instability when the plasma density exceeds a certain critical value. Within that scenario, the ensemble of oscillators sitting in the external magnetic field is no longer held together by the field. The effect should be taken into account in applications related to plasmas of a relatively high density.*

Keywords*: cyclotron radiation; cyclotron resonance; particle dynamics; plasma, synchronization.*

Introduction

Cyclotron resonances are among the best known and most extensively studied resonant effects. They find widespread application, as exemplified by such devices as the cyclotron resonance maser (CRM) and gyrotrons. Despite comprehensive investigations of particle-wave interactions under cyclotron resonance conditions, many unexplored facets still re-

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main, concerning the dynamics of energy exchange between particles and waves during cyclotron resonances.

The results presented in papers [1, 2] show, among other things, that a correct formulation of the cyclotron resonance conditions requires considering the strength of the wave which the particles interact with. Taking that into account may dramatically change our picture of particle dynamics, in particular one can expect significant changes in the phase dynamics of particles.

In this paper, we conduct a study on the phase synchronization of electrons in an ideal plasma subjected to a uniform, d. c. external magnetic field. Our investigation is focused on two types of synchronization, specifically the synchronization induced by an external electromagnetic field, and such produced by the cyclotron radiation emitted by the electrons. Our findings reveal that an external electromagnetic wave can steer particles towards phase synchronization, thus leading to formation of phased bunches. This method of particle grouping demonstrates potential for a shorter synchronization time – as compared with the known mechanisms exploiting relativistic effects [3, 4].

In contrast to the many prior studies that ignored the low-level radiation emitted by charged particles in an external magnetic field, we have included these effects in the analysis. As has been shown, the cyclotron radiation leads to self-synchronization of the electron phases. This radiation can significantly influence the dynamics of the ensemble of charged particles, with the appearance of low-frequency natural vibration modes. Furthermore, at a sufficiently high density of charged particles in the ensemble, an instability can develop. Such instability might be able to disrupt the ensemble. Here, we suggest an estimate of the additional fluctuations capable of undermining the synchronization process.

The paper is organized as follows. In Section 1, it is shown that an external electromagnetic wave can direct particles towards phase synchronization, which results in formation of phase bunches. This mechanism of particle grouping could potentially be more significant, from the point of the synchronization time, than other known mechanisms because of the relativistic velocities, as seen by the examples of the CRM and gyrotrons [3, 4].

Section 2 demonstrates that the plasma particles (electrons) moving through an external magnetic field can emit waves of cyclotron frequency, which effect leads to phase self-synchronization. Under such circumstances, collective oscillations of the particles can appear, in the form of low-frequency eigenmodes. Furthermore, an instability can develop that might be capable of disrupting the magnetic confinement of the plasma.

In Section 3, we discuss how fluctuations can affect the synchronization process and particle dynamics. An estimate of the fluctuation level required for inhibiting the synchronization process is provided. In conclusion, we summarize our findings.

1. Phase grouping of an ensemble of particles

Consider the dynamics of an arbitrary particle moving within a uniform, d. c. magnetic field $\vec{H}_0 = \{0, 0, H_0\}$ in the presence of an electromagnetic wave with transverse field components as follows

$$E_x = E\sin\psi,\ E_y = E\cos\psi.$$

Here $\psi = \omega_H t + \varphi$, where ω_H denotes the cyclotron rotation frequency of a particle in the magnetic field, and φ is the wave's phase. The wavelength λ is assumed to be greatly in excess of the electron's Larmor radius r_L. Consequently, the transverse-plane nonrelativistic dynamics of the particles in such fields can be described by the equations

$$\dot{\upsilon}_{xk} = \upsilon_{yk} + \varepsilon \cdot \sin\psi,\ \dot{\upsilon}_{yk} = -\upsilon_{xk} + \varepsilon \cdot \cos\psi, \qquad (1)$$

with $\tau = \omega_H t$ and $\dot{\upsilon} \equiv d\upsilon / d\tau$, where $\upsilon \equiv v / c$ is the particle speed normalized to the speed of light, finally $\varepsilon = eE / mc\omega_H$ is the wave's "strength parameter" (nonlinear parameter).

Note that the phase of the external wave does not show an explicit dependence on the spatial coordinate, because of $\lambda \gg r_L$. If the external field were neglected i.e., $(\varepsilon = 0)$, then the solutions of Eq. (1) could be represented as

$$\upsilon_{kx} = A_k \sin(\tau + \varphi_k),\ \upsilon_{ky} = A_k \cos(\tau + \varphi_k), \qquad (2)$$

with

$$\{A_k, \varphi_k\} = const.$$

With an external field taken into account (i.e., $(\varepsilon \neq 0)$), the amplitude of the particle's velocity and its phase relative to that of the wave both become time-dependent, and $\{A_k, \varphi_k\}$ is no more constant. Then we find from Eq. (2)

$$\dot{v}_{x,k} = A_k \cos(\tau + \varphi_k) + \varepsilon \sin(\tau + \varphi). \quad (3)$$

Next, we will assume that the wave strength parameter ε is small. In this case, the derivatives of the amplitude and the phase will also be small. Upon equating quantities of the same order we arrive at

$$A_k \cos(\tau + \varphi_k)[\dot{\varphi}_k] + \dot{A} \sin(\tau + \varphi_k) = \\ = \varepsilon \sin(\tau + \varphi). \quad (4)$$

Upon multiplying both sides of Eq. (4) by $\cos(\tau + \varphi_k)$ and averaging we obtainby using Eq. (3) a set of coupled equations determining the dynamics of the phase and the amplitude, specifically

$$\dot{\varphi}_k = \left(\frac{\varepsilon}{A_k}\right) \sin(\varphi - \varphi_k), \\ \dot{A}_k = \varepsilon \cos(\varphi - \varphi_k). \quad (5)$$

Let us also pay attention to the fact that the amplitude is small, i.e. $A_k = v_k / c \ll 1$, and varies with time at a much slower rate than the phase. This enables us to consider the amplitude as a constant magnitude in the equation for the phase. This equation can be conveniently re-written as

$$\dot{\Delta}_k = -\Gamma_k \cdot \sin \Delta_k, \quad (6)$$

where $\Delta_k = \varphi - \varphi_k$ and $\Gamma_k = \varepsilon / A_k$. The singular points of Eq. (6) are $\Delta_{k0} = \pi n$, where $n = 0, 1, 2, \ldots$. The states corresponding to even values of n are stable, whereas odd n correspond to unstable states. The trajectories either approach to or leave the areas of stable (or unstable) states of Eq. (6) as

$$\Delta_k = \Delta_k(0) \exp\left(\pm 2\Gamma_k \tau\right). \quad (7)$$

The upper sign corresponds to unstable, while the lower one to stable states. Thus, all the particles of a group bear the same phase. An ensemble like this demonstrates a particularly efficient interaction with the electromagnetic wave.

Assuming that the phase has reached one of the states like $\Delta_{k0} = \pi n$, we can find, from the equation for A_k in the set Eqs. (5), the behavior of the velocity amplitude

$$A_k = \pm \varepsilon \tau + C, \quad (8)$$

where the plus and the minus signs correspond, respectively, to stable and unstable states (C is a constant magnitude). An unlimited increase or decrease of A_k (velocity), if occurring according to Eq. (8), arises from a lack of account for the energy losses owing to radiation by the particles, else because of nonlinear effects.

Let us compare the grouping mechanism of Eq. (7) with the conventional mechanism of particle grouping in the CRM (see, for example, paper [4]). The change in the particle phase can be expressed as

$$\frac{d\varphi}{dt} = \frac{\omega_H}{\gamma}, \quad (9)$$

where

$$\gamma = (1 - v^2)^{-1/2}.$$

If the particles have different energies, then any change in the phase separation between them shall depend on the difference between these energies, viz.

$$\Delta_{21} = \Delta_0 - \Delta_\gamma \cdot \tau_\gamma. \quad (10)$$

Here $\Delta_{21} = \varphi_2 - \varphi_1$, τ_γ is the synchronization time; Δ_0 the maximum phase separation between the particles, and Δ_γ stands for variation of the parameter γ resulting from a change in the particle's energy. If the particles are synchronized, which means $(\Delta_{21} = 0)$, then from Eqs. (9), (10) we can get an expression for the synchronization time, $\tau_\gamma = \Delta_0 / \Delta_\gamma$.

Let us compare this synchronization time with such following from Eq. (7), namely

$$\tau_E \approx 1/(2\Gamma) = v/(2\varepsilon).$$

To estimate τ_γ, it is necessary to determine the magnitude of the particles' energy change during their interaction with the wave. Also, we believe the grouping to occur when collective processes have not yet manifested themselves. In this case $\Delta_\gamma \approx \approx (v^2 - v_0^2)/2 \approx (v \cdot \Delta v) \ll 1$, which suggests

$$\Delta v \approx \varepsilon \tau_\gamma \text{ and } \tau_\gamma \approx \sqrt{\Delta_0 / \varepsilon v}.$$

Also, we select such a pair of particles where one would be in a stationary (stable) phase. Its phase position is not changed and its energy remains constant. The other particle is accelerated to a maximum possible degree. By following this procedure, we obtain an estimate, as given below, for the synchronization times ratio:

$$\frac{\tau_E}{\tau_\gamma} \approx \upsilon\sqrt{\frac{\upsilon}{\varepsilon}}.$$

Hence, if the inequality $\varepsilon > \upsilon^3$ holds, then the synchronization mechanism of Eq. (7) that we have considered proves to be more efficient than the known mechanism based on relativistic effects.

2. Effect of cyclotron radiation on plasma dynamics

2.1. Self-synchronization of rotating particles

As has been shown in the previous Section, an external electromagnetic radiation does synchronize the phases of rotating charged particles. Here we will demonstrate that similar synchronization owes to the particle interaction effectuated solely through cyclotron radiation, without any external forcing. A well-known example of such synchronization is the coupled clocks effect, first described by Huygens.

Once again, we will consider an ensemble of non-relativistic charged particles (electrons) placed in a d. c. magnetic field $\vec{H}_0 = \{0, 0, H_0\}$. The rotation of and radiation from the electrons in such a magnetic field both take place at the cyclotron frequency $\omega_H = eH / mc$ independent of the electrons' speed. The radiation produces an electric near-field about each of the particles, which is of intensity [5]

$$\vec{E} = -\frac{e\dot{\vec{\upsilon}}}{c^2 R}. \tag{11}$$

Here *e* is the charge of the particle, $\dot{\vec{\upsilon}}$ the particle's acceleration during its rotation, and *R* stands for separation between the particles.

Eq. (11) describes the electric field strength of a rotating particle at a close distance $R \le \lambda$ from that. The particles can interact with each other via this radiation. Note that the field strength at a distance, say, $R \sim \lambda \sim 10$ cm from the radiating particle is extremely small, namely $E \sim 10^{-13}$ V/cm. Usually, such fields are not taken into consideration when discussing plasma dynamics. However, the following analysis demonstrates that such fields can have a significant impact on plasma dynamics, particularly, by provoking phase synchronization between the electrons rotating in the magnetic field.

We start by considering the interaction of two non-relativistic particles, focusing only on transverse motion of the particles with respect to the magnetic field. The dynamics of one of such particles can be described by the equations as follows,

$$\begin{aligned} \dot{\upsilon}_{x1} &= \omega_H \upsilon_{y1} - \mu \dot{x}_{x2}, \\ \dot{\upsilon}_{y1} &= \omega_H \upsilon_{x1} - \mu \dot{x}_{y2}, \end{aligned} \tag{12}$$

where $\mu = e^2 / Rmc^2$ is the coupling coefficient between the particles.

Since the coupling coefficient is small, the unperturbed values of the fields radiated by the particles can be substituted into the last terms on the right-hand side of Eq. (12). The transverse components of the field emitted by the particles are $E_{xj} = E_j \sin\psi_j$ and $E_{yj} = E_j \cos\psi_j$, where $\psi_j = \omega_H t + \varphi_j$. Here the wavelengths λ are much larger than the Larmor radius of the electrons $(\lambda \gg r_L)$. Therefore, we have been able to neglect the spatial dependences exhibited by phases of the waves emitted. Thus, the equation set Eq. (12) can be easily generalized for an arbitrary number of particles $N (N \gg 1)$:

$$\begin{aligned} \dot{\upsilon}_{xk} &= \upsilon_{yk} - \sum_{j=1, j\ne k}^{N} \varepsilon_j \cdot \cos\psi_j, \\ \dot{\upsilon}_{yk} &= -\upsilon_{xk} - \sum_{j=1, j\ne k}^{N} \varepsilon_j \cdot \sin\psi_j. \end{aligned} \tag{13}$$

Here $\varepsilon_j = \mu_j A_j$; $\mu_j = e^2 / R_{kj} mc^2$; $\psi_j = \omega_H t + \varphi_j$; $\tau = \omega_H t$; $\dot{\upsilon} = d\upsilon / d\tau$; $A_j = eE_j / m\omega_H$.

The sums present in the right-hand sides of Eqs. (13) take into account the effect of other particles on the dynamics of an arbitrary particle subscribed with *k*. Should that effect be neglected, for example by setting $\varepsilon_j = 0$, the solutions of Eqs. (13) would read

$$\upsilon_x = A\sin(\tau + \varphi) \text{ and } \upsilon_y = A\cos(\tau + \varphi), \tag{14}$$

with

$$\{A, \varphi\} = const.$$

Making account of the radiated fields we arrive at $\varepsilon_j \ne 0$, which leads to the appearance of a time dependence of both the amplitude and phase, i.e., $\{A, \varphi\} \ne const$. In this case, Eqs. (13) and (14) permit obtaining relationships between the amplitudes,

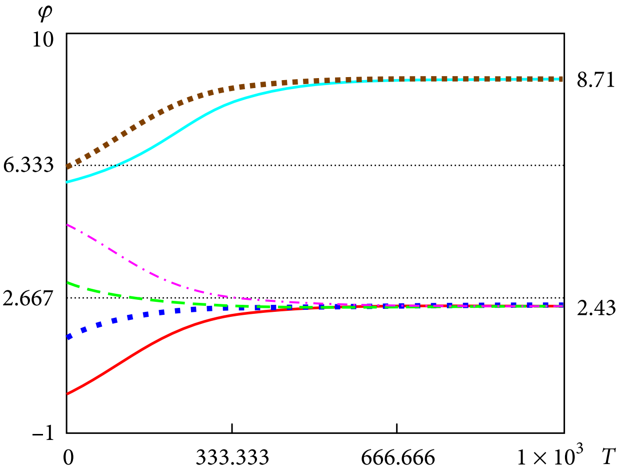

Fig. 1. Time-dependent phases of interacting oscillators, $N = 10$

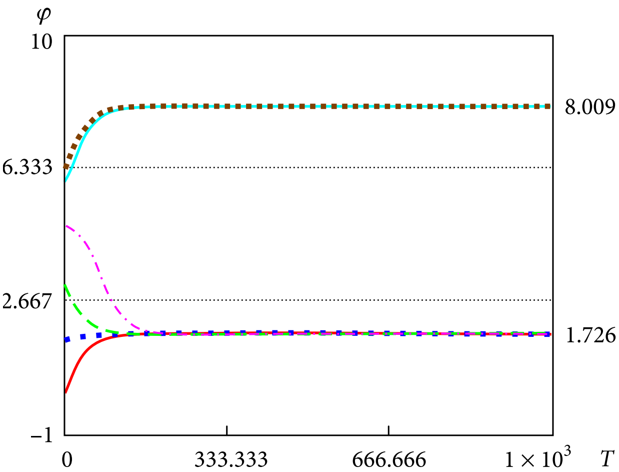

Fig. 2. Time-dependent phases of interacting oscillators, $N = 60$

phases and their derivatives, like

$$\dot{v}_{x,k} = A_k \cos(\tau + \varphi_k)[1 + \dot{\varphi}_k] + \dot{A} \sin(\tau + \varphi_k) =$$

$$= A_k \cos(\tau + \varphi_k) + \sum_{j \neq k}^{N} \varepsilon_j \sin(\tau + \varphi_j). \qquad (15)$$

Upon multiplying Eq. (15) (first, by $\cos(\tau + \varphi_k)$ and then by $\sin(\tau + \varphi_k)$), we can obtain, after proper averaging, a set of equations to describe the dynamics of particles' phases and amplitudes,

$$\dot{\varphi}_k = \frac{1}{A_k} \sum_{j \neq k} \varepsilon_j \sin(\varphi_j - \varphi_k),$$
$$\dot{A}_k = \sum_{j \neq k} \varepsilon_j \cos(\varphi_j - \varphi_k). \qquad (16)$$

Like in Eq. (5), the amplitudes A_j show a much slower rate of variations as compared with the phases φ_j. Once again, this allows us considering A_j, with $j = 1, 2 \ldots$ as constant magnitudes in the first equation of the set Eqs. (16).

Considering the A_j and μ_j as sets of N statistically independent random values, we may transform the equation for the phases into

$$\dot{\varphi}_k = \varepsilon_N \operatorname{Im}\left\{ e^{-i\varphi_k} \frac{1}{N} \sum_{j \neq k} e^{i\varphi_j} \right\} =$$

$$= \varepsilon_N \operatorname{Im}\left\{ e^{i(\varphi_k - \varphi_j)} \frac{1}{N} \sum_{j \neq k,l} e^{i(\varphi_j - \varphi_l)} \right\}. \qquad (17)$$

Here $\varepsilon_N = N \langle \mu \rangle \langle A \rangle / A_k$, where $\langle \mu \rangle$ and $\langle A \rangle$ denote average values of μ_j and A_j, respectively.

This relation makes it possible to obtain an equation as follows for finding the separation between the phases of two arbitrary particles,

$$\dot{\Delta}_{kl} = \varepsilon_N \Big\{ \cos(\Delta_{kl}) \Big[\langle \sin(\Delta_{jl}) \rangle - \langle \sin(\Delta_{jk}) \rangle \Big] -$$
$$- \sin(\Delta_{kl}) \Big[\langle \cos(\Delta_{jl}) \rangle + \langle \cos(\Delta_{jk}) \rangle \Big] \Big\}. \qquad (18)$$

Here $\Delta_{kl} = \varphi_k - \varphi_l$, $\langle \exp(i\varphi) \rangle = \frac{1}{N} \sum_{j \neq k}^{N} \exp(i\varphi_j)$.

It is easy to see that the functions Δ_{kl} tend to zero, while they are the relations which the equation set Eqs. (17) holds for. In addition, with $\Delta_{kl} \to 0$ the first term in Eq. (18) is proportional to $(\Delta_{kl})^3$, while the second one is proportional to Δ_{kl}. Therefore, in order to determine the time dependence of Δ_{kl}, it is sufficient to keep only the latter terms in the right-hand part of Eq. (18), viz.

$$\dot{\Delta}_{kl} = -2\varepsilon_N \sin \Delta_{kl}. \qquad (19)$$

Equations (19) possess singular points at coordinates $\Delta_{kl} = n\pi$ where n is an arbitrary integer. The states corresponding to even values of n are stable, their synchronization will always take place in cases of an initially non-uniform phase distribution. The states with odd values of n are unstable.

The behavior of the phases in the vicinity of the singular points is

$$\Delta_{kl} = \Delta_{kl}(0) \exp(\pm 2\varepsilon_N \tau). \qquad (20)$$

The upper and the lower signs correspond to unstable and stable states, respectively.

The process of the synchronization of the electrons rotating in an external field, described in Section 2, resembles such for the case of self-synchronization. However, in the latter case, the characteristic time required for the synchronization is, according to Eq. (20), $1/(2\varepsilon_N\omega_N)$, with $\varepsilon_N \sim N$. For a given cyclotron frequency, this time decreases as the electron density *n* is increased.

The analytical results presented above are in a good agreement with our numerical study of the initial relations Eqs. (16). The results of numerical simulations are shown in Figs. 1 and 2 for a total number of interacting particles equal to 10 and 60, respectively. These figures present time dependences of phases of the six particles possessing initial phases $\varphi(0)$ equal to $0; \pi/2; \pi; 3\pi/2; 7.5\pi/2$ and 2π. In these simulations, the relationship between their intrinsic time and the laboratory time is determined by the formula $t = T_n/(\omega_H \varepsilon_N)$. As can be observed from Figs. 1 and 2, the phases tend to two stationary values which differ by 2π. From the physical point of view, the self-synchronization has occurred with the phases of the steady state remaining unchanged. In addition, comparison of the figures has revealed that an increase in the number of interacting particles leads to a decrease in the synchronization time, as indicated by Eq. (20).

2.2. Stability Analysis

The results obtained above show that an ensemble of charged particles (electrons) placed in an external magnetic field tends to phase-lock individual particles of the ensemble. Practically, the ensemble considered can be seen as a model of an ideal plasma placed in a magnetic field. Therefore, it seems of interest to investigate the consequences of the phase synchronization of individual plasma particles. Below, we will show that the phase synchronization of particles can radically change the dynamics of the particle ensemble considered.

To study the dynamics of an ensemble of phased particles, we start from their equations of motion, Eq. (12). By differentiating this equation set and performing some standard transformations, we arrive at the equations as follows to describe the dynamics of just two interacting particles,

$$\ddot{v}_{x1} + \omega_H^2 v_{x1} = 2\mu\omega_H^2 v_{x2},$$
$$\ddot{v}_{y1} + \omega_H^2 v_{y1} = 2\mu\omega_H^2 v_{y2}.$$

For an arbitrary number of particles, the set of equations to describe the dynamics of the velocity components can be represented as

$$\ddot{v}_k + v_k = 2\sum_{j=1,\, j\neq k}^{N} \mu_j v_j. \tag{21}$$

Here, the variables v_k determine either the *x*-component or the *y*-component of the *k*-th particle velocity. Also, the dimensionless time $\tau = \omega_H t$ is used. The coupling coefficients μ_j differ from one another only by the amount of separation R_j between the particles.

Note that for the case of many oscillators $(N \gg 1)$ (and small coupling coefficients $(\mu_j \ll 1)$, the right-hand side of Eq. (21) remains the same for oscillators of all kinds (having different values of the index *k*). In this case, Eq. (21) can be significantly simplified, viz.

$$\ddot{v}_k + \left(1 - 2\sum_{j=1}^{N}\mu_j\right)v_k = 0. \tag{22}$$

Thus, dynamics of the ensemble is completely determined by the sum of the coupling coefficients. To interpret this condition in terms of physical parameters, let us calculate the sum in Eq. (22). The summation should be performed over coupling coefficients of the particles located in a volume of characteristic dimension $\lambda/2$. This limitation of the volume stems from the initial assumption concerning the possibility of neglecting the spatial variation of the phase of the particle-radiated wave. The summation can be replaced by integration over a sphere of radius $\lambda/2$. With account of $\mu_j = e^2/(R_j mc^2)$, the integration yields

$$\sum_{j=1}^{N}\mu_j = \frac{\pi n\lambda^2 e^2}{2mc^2}. \tag{23}$$

Thus, in accordance with Eqs. (22) and (23) the natural frequency of the ensemble of electrons, specifically

$$\omega_n = \sqrt{1 - \frac{\pi n\lambda^2 e^2}{mc^2}},$$

decreases as the electron density *n* is increased. This results in low-frequency oscillations appea-ring in the plasma. In case the density *n* exceeds the threshold value, such that

$$n > \frac{mc^2}{\pi\lambda^2 e^2}, \quad (24)$$

the plasma dynamics goes unstable. The critical density is determined solely by the cyclotron wavelength λ and, accordingly, the d. c. magnetic field's strength. For instance, the wavelength λ = 3 cm corresponds to a critical density about 10^{11} cm^{-3}. Should the condition of Eq. (24) be satisfied, the particles would stop their regular rotation in the magnetic field. The speed of their motion becomes unstable and the particles start being accelerated. Of importance is the fact that the particles are no longer retained by an external magnetic field. This is the case of a radical change in particle dynamics.

3. Suppression of phase synchronization

Thus, within the framework of the model we have just considered, an ensemble of charged particles becomes unstable when the density of oscillators exceeds some critical value. The question arises about a possible mechanism to suppress the instability. From a general standpoint, a sufficient level of fluctuations may suppress the phase synchronization. In fact, this might prove to be a rather difficult problem for being trea-ted analytically. However, estimates for the required magnitude of fluctuations are easily obtainable. To do that, we add a term in the right-hand side of Eq. (17) for the phase, representing additive random forces, i.e.

$$\varphi_k = \varepsilon_N \,\mathrm{Im}\left\{ e^{i(\varphi_k-\varphi_i)} \frac{1}{N} \sum_{j\neq k,l} e^{i(\varphi_j-\varphi_l)} \right\} + \xi(\tau). \quad (25)$$

That addendum in the right-hand side of Eq. (25) can be assumed, for the sake of simplicity, to be a delta-correlated random function with the diffusion coefficient D, viz.

$$\langle \xi(t) \rangle = 0; \langle \xi(t)\xi(t) \rangle = D\delta(t-t_1).$$

It is possible to obtain solutions to the equations Eq. (25), however, their form may happen to be cumbersome. Meanwhile, it is easy to obtain an estimate for the value of the diffusion coefficient D that would be sufficient for suppressing the phase synchronization. To do this, one needs to compare two processes, namely one without fluctuations ($D = 0$) and the other with fluctuations, hence $D \neq 0$. The solution of Eq. (25) for the latter case and with $\varepsilon_N = 0$ is well known, namely the r.m.s. level of phase fluctuations under the action of a random force reads as

$$\sigma = \sqrt{\langle \varphi_i^2 \rangle} = D\sqrt{2\tau}.$$

The diffusion time T over which the phase is changed by 2π (in fact $\sigma = 2\pi$) can be estimated as $T \approx 2\pi^2 / D$. During this time, the phases become separated, because of the synchronization process, by

$$\Delta = \pi \cdot \exp(-(2\varepsilon_{lN})T),$$

where

$$\varepsilon_{lN} = \frac{1}{A_l}\sum_{j\neq l}^{N} \mu_{lj}A_j \sim N\mu = \frac{e^2 \cdot N}{mc^2 R}.$$

If the inequality $(2\varepsilon_N)T \ll 1$ holds, the synchronization process is subject to suppression by diffusion.

The synchronization process should prevail provided that the number of oscillators were greater than the following critical value

$$N > \frac{1}{2\mu T} = \frac{D}{4\pi^2 \cdot \mu} \sim DR \cdot 10^{11},$$

where R is the average separation between the particles.

Conclusions

1. The most important result is the predicted appearance of low-frequency oscillations in an ensemble of charged oscillators, with a further onset of plasma instability development in case the plasma density exceeds a certain critical value. Under conditions of the instability, the ensemble of oscillators moving in an external magnetic field can no longer be confined by that field.

This result is important for two reasons. First, the electromagnetic field resulting from cyclotron radiation of the oscillators, acts as a coupling agent between the oscillators. While the field produced by a single electron is of low intensity, the great number of phase-locked oscillators do generate a field that can offer noticeable effects. Second, while the dynamics of an arbitrary number of oscillators has been well studied in previous works (see, for instance, [6, 7]), the oscillator frequencies and the coupling between them were assumed to be determined by the same potential in which the charged particles moved. In our case, the coupling coefficients are determined by another physical mechanism, namely, the one associated with the radiation from the oscillators themselves.

As has been expected, both the self-synchronization effect and the instability that follows are suppressible in the presence of an external fluctuating field. We have suggested an estimate for the intensity of delta-correlated random fluctuations that might be required for undermining the synchronization process and that of plasma dynamics stabilization.

2. The phase synchronization of particles in the CRM devices and gyrotrons is an essential physical process. This synchronization is determined by relativity effects [3, 4]. The paper considers the phase synchronization, the result of which is that the particles get grouped with such phases where the wave delivers energy to these particles. Consequently, this mechanism can restrict the level of the excited field in CRM devices and gyrotrons.

3. It should be noted that the model considered in this work is not fully self-consistent. First, this is due to the fact that the fields considered in the second section are determined by the radiation of individual unphased oscillators. The radiation in the form of transverse electromagnetic waves leaves the region occupied by the oscillators. These losses are not taken into account in the model. Additionally, an increase in the strength of the field excited by the particles at cyclotron resonances may lead to regimes with dynamic chaos (see, for example, [1, 2, 8]). These issues are supposed to be considered in a future work.

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В.О. Буц[1,2], *Д.М. Ваврів*[2]

[1] Національний науковий центр
«Харківський фізико-технічний інститут» НАН України
вул. Академічна, 1, м. Харків, 61108, Україна
[2] Радіоастрономічний інститут НАН України
вул. Мистецтв, 4, м. Харків, 61002, Україна

ФАЗОВА СИНХРОНІЗАЦІЯ ЧАСТИНОК ПРИ ЦИКЛОТРОННИХ РЕЗОНАНСАХ

Предмет і мета роботи. Досліджено фазову синхронізацію електронів в ідеальній плазмі в постійному однорідному зовнішньому магнітному полі. Ми розглядаємо дві форми синхронізації: синхронізацію за допомогою зовнішнього електромагнітного поля та синхронізацію за допомогою циклотронного випромінювання, що випускається електронами. Наша мета — порівняти ці форми синхронізації та їхній вплив на стабільність плазми.

Методи та методологія. Плазму показано як сукупність пов'язаних осциляторів, динаміка яких описується набором пов'язаних диференціальних рівнянь. Враховуючи невеликий зв'язок між осциляторами, ми знаходимо аналітичний розв'язок рівняння та виконуємо аналіз стійкості, використовуючи стандартні підходи теорії динамічних систем. Ці розв'язки підтверджено відповідним чисельним моделюванням.

Результати. Ми демонструємо, що зовнішня електромагнітна хвиля може направляти частинки до фазової синхронізації, що призводить до утворення фазових згустків. Цей механізм групування частинок може бути більш ефективним з точки зору часу синхронізації порівняно з відомими механізмами, заснованими на теорії відносності. Крім того, ми показуємо, що циклотронне випромінювання, випущене зарядженими частинками, яке часто не враховується через його малість, призводить до самофазової синхронізації електронів. До того ж, якщо щільність заряджених частинок в ансамблі досить висока, може виникнути нестабільність, що потенційно може порушити ансамбль. Ми надаємо оцінку випадкових флуктуацій, необхідних для зриву процесу синхронізації та стабілізації динаміки плазми.

Висновки. Найбільш значущим результатом є виникнення низькочастотних коливань у сукупності заряджених осциляторів, що супроводжується появою нестабільності плазми, коли густина плазми перевищує критичне значення. У такому сценарії ансамбль осциляторів у зовнішньому магнітному полі більше не утримується разом полем. Цей ефект слід враховувати в розробках, пов'язаних з використанням плазми відносно високої щільності.

Ключові слова: *циклотронне випромінювання, циклотронні резонанси, динаміка частинок, плазма, синхронізація.*