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Drift Instability In Flows of Dusty Plasma with Continuous Size Spectrum of Grains

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The instability of electrostatic disturbances in flows of dusty plasma is analyzed for the different types of size (mass) spectrum of dust components. It is shown that any kind of the spectrum leads to the growth of the longitudinal waves. Only quadratic spectrum is an exception. In this case the eigenwaves of dusty plasma are stable.

Introduction

Charged dust grains are often encountered in space (e.g. in planetary rings, comet tails, interstellar dust clouds, etc.). If the dust-particle density is sufficiently high, these grains, along with electrons and ions, are involved in collective processes and form a mixture that is referred to as a dusty plasma [1]. At first sight it would seem that there is no principal distinction between a dusty and a conventional multicomponent plasma with different sorts of ions. Really, the dispersion equation written in hydrodynamical approach for the electrostatic waves in electron-ion plasma

$$1 = \sum_{\alpha} \frac{\omega_{p,\alpha}^2}{\left(\omega - kv_{0,\alpha}\right)^2 - \gamma\, k^2 v_{T,\alpha}^2} \qquad (1)$$

is also valid for a dusty plasma if the summation over α is extended to all species of microparticles (electrons $\alpha = e$, ions $\alpha = i$) and dusty grains ($\alpha = 1,2,\dots,N$, where N is the number of grains sorts). We use the standard symbols: $\omega_{p\alpha} = \left(4\pi q_\alpha^2 n_{0,\alpha}/m_\alpha\right)^{1/2}$ is plasma frequency, $v_{T,\alpha}$ - the thermal velocity, $q_\alpha, m_\alpha, n_{0\alpha}, v_{0\alpha}$ are the charge, mass, "background" density and drift velocity of the particles of species α respectively, ω is the frequency and k is the wavenumber (all the quantities vary as $\exp\left(-i\omega t + ikr\right)$). The coefficient γ in (1) is determined by the isentropic exponent in the state equation of plasma.

Nevertheless, the waves in dusty plasma possess many special features. We indicate some of them. At first, there are great variations in the numerical values of the ratio q_α/m_α for the dusty grains in comparison with ions and electrons. Because of this, if the wave frequency ω is sufficiently high, the grains may be considered as fixed ones in the media of oscillating microparticles. However, even in this case the presence of dust grains in plasma can affect the properties of eigenwaves. The matter is that the charged grains create an inhomogeneous electric field, which may be represented as a wave with a "zero" frequency. When the necessary resonance condition resulting from interaction of the propagating wave and the above-mentioned standing wave is fulfilled, nonlinear Landau damping can occur [2].

The distinctive features of dusty plasma manifest themselves more markedly in the low-frequency range when dust grains are involved in wave motion. In this case, we must take into account the fact that the charge on the grains, unlike that of the electrons and ions, can change. During the propagation of longitudinal waves, the variation of the grain charge appears with a delay due to the finite electric capacitance of the grain. As a result, an additional attenuation mechanism that is absent in ordinary plasma arises [3].

Another feature of dusty plasma is connected with continuous distribution of the dusty grains over sizes (masses). So far, only one or several sorts of particles were considered. But the assumption of a rarefied spectrum of grain sizes is unlikely to be true under real conditions. Moreover, the results of space-plasma observations indicate that, in some range of sizes $R_{\min} \le R \le R_{\max}$ the distribution over the grain sizes $W(R)$ is continuous [4]. It was shown that in this case a slow electrostatic wave, named dust acoustic, acquires collisionless attenuation (analog of Landau damping) even in hydrodynamic approach [5]. The situation may be changed if the relative motions of the grains and microparticles will be taken into account. As will be shown below, such difference of the drift velocities usually exists in plasma in the vicinity of magnetized planet. It leads to arising analogs of kinetic instabilities in flows of dusty plasmas independently of the $W(R)$ shape. Only quadratic distribution function will be stationary.

2.Instability of the Wave Disturbances in Flows of Dusty Plasma

We turn to equation (1) to consider the electrostatic waves, assuming all equilibrium velocities of the particles $v_{0\alpha} \neq 0$, while the thermal velocities $v_{T\alpha} = 0$ (another limiting case $v_{T\alpha} \neq 0, v_{0\alpha} = 0$ is analyzed in detail, in [5]). If the spectrum of grain sizes is continuous, then summation over all sorts of grains in (1) should be replaced by integration with substitution instead of the $\omega^2_{p\alpha}$ the dimensional value

$$\omega^{*2}_p = 4\pi q^2(R)W(R)/m(R) = $$
$$= 3\varphi^2 W(R)\rho^{-1}R^{-1}$$

(where $\varphi \cong q(R)/R$ is the grain potential, it is often determined by the plasma temperature alone, being independent of grain size $R[l]$; $m(R) \cong \frac{4}{3}\pi\rho R^3$ are the masses , and ρ is the mass density of the grain material). The dispersion relation (1) becomes

$$\varepsilon(\omega,k) = 1 - \sum_{\alpha=e,i}\frac{\omega^2_{p\alpha}}{(\omega - kv_{0\alpha})^2} + \int_{R_1}^{R_2}\frac{\omega^{*2}_p(R)dR}{(\omega - kv_0(R))^2} = 0 \qquad (2)$$

(note, that the dimensionality of value $\omega^{*2}_p(R)$ is $[\omega^{*2}_p(R)] = (c^{-2}cm^{-1})$). Due to continuous dependencies of particle parameters $q(R), m(R)$ and $v_0(R)$ there appears a term characteristic for the classical kinetic theory equations with the velocity distribution function $f_0(v_0)$. Actually, the integral in (2) can be represented in the form

$$J = \int_{-\infty}^{\infty}\frac{f_0(v_0)dv_0}{(\omega - kv_0)^2}, \qquad (3)$$

where

$$f_0(v_0) = \begin{cases}\omega^{*2}_p(R(v_0))dR/dv_0, & v_1 = v_0(R_{min}) < v_0 < v_0(R_{max}) = v_2, \\ 0, & v_0 < v_1, v_0 > v_2\end{cases} \qquad (4)$$

is equivalent distribution function. Further investigation of modal equation (2) can be performed just as it is done in kinetic theory. However, the final results depend on the specific functions of $W(R)$ and $v_0(R)$. According to literature data, $W(R)$ is often a power-law function $\sim R^{-\mu}$ with the μ lying between 0.9 and 4.5 [4]. We will evaluate the integral (2) for the distribution function $W(R) = N_0R_0^{\mu-1}/R^{\mu}$. It remains only to obtain the dependence $v_0(R)$.

Space plasmas often involve particle streams with ordered velocities. Consider, for example, the dusty plasma in the vicinity of a magnetized planet. The neutral particles move through the gravitational field of the central body in accordance with Kepler's laws. Contrary to this, the motion of microparticles (i.e. electrons and ions) is governed by electromagnetic forces and they corotate with the planet. As for the electrically charged dust grains, they "feel" both gravitational and electromagnetic forces. As a result, such particles do not move around the planet at Kepler velocity v but rather at somewhat different velocity v_0 which is determined by the charge/mass ratio. In particular, the linear velocity of grain moving along an equatorial-plane circular orbit of radius r is [6]

$$v_0 \cong v\left[1 + \frac{\Omega_B}{2\Omega_k^2}(\Omega_k - \Omega_p)\right]. \qquad (5)$$

Here Ω_k is the Kepler frequency, $\Omega_B = \frac{q(R)B_0}{m(R)c}$ is the gyrofrequency, B_0 and Ω_p are the planetary magnetic field and rotation frequency, respectively; c is the velocity of light. Taking into account the dependence $\Omega_B(R)$ we can write

$$v_0(R) \cong v(1 \pm l^2/R^2) \qquad (6)$$

with $l^2 = \left|\frac{3B_0\varphi}{4\pi\rho c\Omega_k^2}(\Omega_k - \Omega_p)\right|$. The sign in equation (6) depends on the radial distance from the planet and the sign of the particle's electric charge [6]. For the sake of definiteness, the minus will be adopted. Upon this the boundary velocities v_1 and

v_2 are in the same relation as grains sizes R_1 and R_2 (i.e. $R_1 < R_2$ and $v_1 < v_2$).

Hence, at the orbit of radius r a multistream system exists, which is characterized by the functional dependence (6). Because of this we will investigate the wave disturbances in the thin filament dusty plasma stream with diameter $d << r$. It is an ideal model of the narrow planetary ring. For example, for Saturn's rings the elementary ringlets have $d \sim 10^2 - 10^3 m << r \sim 10^8 m$. It appears that the dispersion relation for the wave disturbances of a relatively short wavelengthes $\lambda << r$ (this is the condition under which the beam curvature can be neglected) takes the same form of equation (2). But in the case of the thin filament beam its parameters are controlled by the linear density of particles $\sim \pi d^2 n_{0\alpha}/4$. It leads to the appearance of the factor $k^2 d^2/32$ before the sum in (2) (this problem considered in detail in the paper [6]). Moreover, since $v_{0e} = v_{0i} = \Omega_p r$, it is convenient to use the frame of reference in which electrons and ions are at rest $\left(v_{0e,i} = 0\right)$, while the grains drift at the velocity (6) only now $v = \left(\Omega_k - \Omega_p\right) r$ and $l^2 = \dfrac{3B_0\varphi}{4\pi\rho c\Omega_k}$.

For the selected dependencies $W(R)$ and $v_0(R)$ the equivalent distribution function (4) becomes

$$f_0(v_0) = \begin{cases} S\left(v - v_0\right)^{(\mu-2)/2}, & v_1 \le v_0 \le v_2; \\ 0, & v_0 > v_2; v_0 < v_1; \end{cases} \qquad (7)$$

with $S = 3\varphi^2 N_0 R_0^{\mu-1} l^{-\mu} v^{-\mu/2} (2\rho)^{-1}$. The final dispersion relation can be written in the form

$$\varepsilon(\omega,k) = 1 - \frac{k^2 d^2}{32}\left[\frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2} + \frac{S}{k}\left[\frac{(v - v_2)^{\frac{\mu-2}{2}}}{(\omega - k v_2)} - \frac{(v - v_1)^{\frac{\mu-2}{2}}}{\omega - k v_1} - \right.\right.$$
$$\left.\left. - \begin{cases} \left[\dfrac{\mu-2}{2}\displaystyle\int \frac{(v - v_0)^{\frac{\mu-4}{2}}}{\omega - k v_0} dv_0\right] = 0, & \omega < k v_1, \omega > k v_2; \\ \left[\dfrac{\mu-2}{2} P.V. \displaystyle\int \frac{(v - v_0)^{\frac{\mu-4}{2}}}{\omega - k v_0} dv_0\right] + i\pi \dfrac{d^2 S}{32} \dfrac{\mu-2}{2}\left(v - \dfrac{\omega}{k}\right)^{\frac{\mu-4}{2}} = 0, & k v_1 \le \omega \le k v_2. \end{cases}\right.\right. \qquad (8)$$

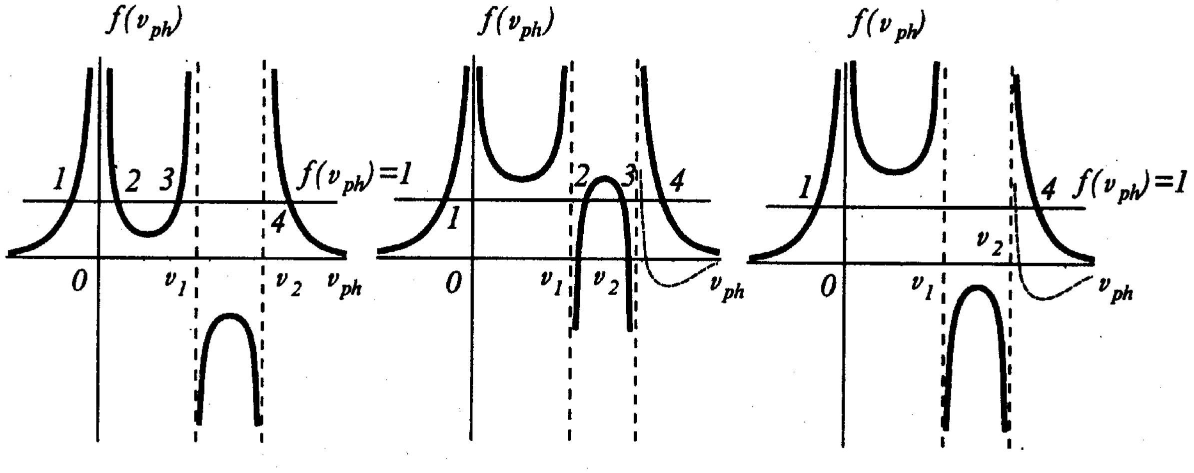

Fig. 1.

The dielectric constant of the dusty plasma becomes complex (in the range of phase velocities $v_1 < v_{ph} < v_2$). Consequently, the wave frequency will be complex also, i.e. $\omega = \omega_r + i\gamma$. It may lead to both the attenuation $(\gamma < 0)$ and the instability $(\gamma > 0)$ of eigenwaves in dusty plasma. We will verify that it leads to the wave instability $(\gamma > 0)$ independently of the particular form of equivalent distribution function $f_0(v_0)$. The exception is $f_0(v_0)$ for $\mu = 2$. It is a peculiar case since the distribution function $f_0(v_0)$ is const to, $\varepsilon_i = 0$ and $\gamma = 0$. The values $\mu > 2$ and $\mu < 2$ are corresponding to the various slopes of $f_0(v_0)$ and hence different signs of $\varepsilon_i(\omega, k)$. The real frequency ω_r is the solution of equation $\varepsilon_r(\omega_r, k)=0$, while γ is given by the expression

$$\gamma \approx -\varepsilon_i(\omega_r, k)/\left(\partial\varepsilon_r(\omega_r, k)/\partial\omega_r\right). \qquad (9)$$

We will analyze these equations along with (8) in three specific cases. The first one is $\mu = 2$. Another two cases $\mu = 4$ and $\mu = 1$ are chosen for the simplicity of the calculations.

$\mu = 2$. The integral term of (8) vanishes and dispersion relation is

$$\varepsilon = 1 - \left(\frac{\omega_{p,e}^2 + \omega_{p,i}^2}{v_{ph}^2} - \frac{\omega_{p,0}^2}{\left(v_{ph} - \overline{v}_0\right)^2 - \Delta v^2} \right) \frac{d^2}{32} = 0, \qquad (10)$$

where $\overline{v}_0 = \frac{1}{2}(v_1 + v_2)$ is the mean velocity and the spread of velocities is $\Delta v = \frac{1}{2}(v_2 - v_1)$.

To analyze (10) let us bring it to the form $f(v_{ph}) = 1$ with $f(v_{ph}) = 1 - \varepsilon(v_{ph})$. The graph is presented in Fig. 1. It is shown three possibilities for $f(v_{ph})$ under various relations between the parameters of the microparticles and dusty grains (see cases a, b, c). Investigating the eigenwave spectrum of dusty plasma, we have restricted ourselves to the peculiar features, connected with dusty component. If the mean drift velocity exceeds the value

$$\overline{v} > \frac{d}{2\sqrt{2}}\left(\omega_{p,e}^2 + \omega_{p,i}^2\right)^{1/2}, \qquad (11)$$

there are two dust-acoustic waves (the roots 3 and 4 in Fig. 1a, respectively)

$$v_{ph}^{(3,4)} \cong \overline{v} \pm \Delta v\left(1 + d^2\lambda_{D,eff}^{-2}\right), \qquad (12)$$

where $\lambda_{D,eff} = \Delta v/\omega_{p,0}$ plays the role of the effective Debye length.

If an inequality opposite the (11) is used and the Δv obeys the condition

$$\Delta v > \frac{\omega_{p,0}}{\left(\omega_{p,e}^2 + \omega_{p,i}^2\right)^{1/2}}\overline{v}, \qquad (13)$$

then (8) has two roots (Fig.1b), which are not far from the boundary velocities v_1 and v_2, i.e.

$$v_{ph}^{(2,3)} \cong v_{1,2}\left(1 \pm \frac{\omega_{p,0}^2 v_{1,2}}{2\left(\omega_{p,i}^2 + \omega_{p,e}^2\right)\Delta v}\right). \qquad (14)$$

Note once more, that the waves (14) are stable.

Finally, the condition of usual beam instability (Fig. 1c) involves an inequality (11) and inequality opposite to (13).

$\mu = 4$ and $\mu = 1$. The equations (8) have got an unwieldy form. Let us turn once again to graphical analysis of these dispersion relations. Only now $f(v_{ph})$ is represented as $f(v_{ph}) = 1 - \varepsilon_r(v_{ph})$. It appears that graphical representations of dependencies $f(v_{ph})$ under $\mu = 4$ and $\mu = 1$ are similar to $f(v_{ph})$ under $\mu = 2$ except the little differences in the region of high phase velocities ($v_{ph} > v_2$). Therefore, the results, obtained for the quadratic spectrum are corresponding to the cases $\mu \neq 2$ qualitatively. Really, the solutions of dispersion equations under $\mu = 4$ and $\mu = 1$ differ from roots of (12) and (14) only by numerical factors.

The eigenwaves with phase velocities $v_1 < v_{ph} < v_2$ are of the most interest for us (see Fig. 1b). Since the dispersion relations include imaginary term, it leads to the damping or excitation of the wave disturbances with $v_{ph}^{(2)}$ and $v_{ph}^{(3)}$.

In the case $\mu = 4$ the imaginary parts of the frequencies $\gamma^{(2,3)}$ are

$$\gamma^{(2)} = +\frac{\pi\omega_{p,0}^4}{4\left(\omega_{pe}^2+\omega_{pi}^2\right)}\frac{kv_1^4}{\Delta v^3}, \qquad (15)$$

and

$$\gamma^{(3)} = -\frac{\pi\omega_{p,0}^4}{32\left(\omega_{pe}^2+\omega_{pi}^2\right)}\frac{kv_2^4\left(v_k-v_2\right)}{\Delta v^4}. \qquad (16)$$

Thus, the wave with phase velocity $v_{ph}^{(2)}$ is growing in time, while the wave $v_{ph}^{(3)}$ is damping.

For the case $\mu = 1$ the situation is opposite. The matter is that under $\mu = 1$, the imaginary part of dielectric constant ε_i has another sign, i.e. $\varepsilon_i < 0$. It leads to the growing of the wave with $v_{ph}^{(3)}$ and damping effect for $v_{ph}^{(2)}$. Notice, that the increment $\gamma^{(3)}$ is negligible small (by analogy with (16) $\gamma^{(3)} \sim k\left(v_k - v_2\right) \to 0$ as the difference of velocities between the neutral and the heaviest grains). It means that for the size spectrum with $\mu \neq 2$ we are concerned only with growing solutions of dispersion equation (2) in the range of phase velocity $\left(v_1 \le v_{ph} \le v_2\right)$. The magnitude μ affects the value of increment γ. In particular, the growith of waves under $\mu > 2$ takes place in a substantially shorter time interval than under $\mu < 2$.

The obtained results are in accordance with the kinetic theory of usual electron-ion plasma [7]. Really, the condition $\partial f_0/\partial v_0 < 0\ (\mu > 2)$ means that resonance particles take the energy from the wave. But the energy of the oscillations $\omega = kv_{ph}^{(2)}$ is negative itself (it is easy to show that $\left(\partial\varepsilon_r\left(\omega_r,k\right)/\partial\omega_r\right)_{kv_{ph}^{(2)}} < 0$. As a result, the increase of the absolute value of the energy and electric field arises. On the contrary, the wave with positive energy attenuated $\left(v_{ph}^{(3)}\right)$. The case $\partial f_0/\partial v_0 > 0\ (\mu < 2)$ corresponds to situation, when grains in resonance give the energy to the wave. It leads to the growth of the wave with a positive energy $\left(v_{ph}^{(3)}\right)$ and attenuation of wave with negative energy $\left(v_{ph}^{(2)}\right)$.

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Дрейфовая неустойчивость в потоках пылевой плазмы с непрерывным распределением частиц по размерам

В.В. Ярошенко

Исследуется неустойчивость электростатических возмущений в потоках пылевой плазмы при различных видах спектра размеров (масс) пылевых частиц. На примере тонкого планетного кольца показано, что независимо от конкретного вида распределения пылинок по размерам, будет наблюдаться неустойчивость продольных волн. Исключение составляет лишь квадратичный спектр - в этом случае собственные волны оказываются стационарными.

Дрейфова нестійкість в потоках пилової плазми з неперервним розподіленням частинок за розмірами

В.В. Ярошенко

Досліджується нестійкість електростатичних хвиль в потоках пилової плазми при різних видах спектру розмірів (мас) пилових частинок. На прикладі тонкого планетного кільця показано, що незалежно від конкретного виду спектру розмірів пилу буде спостерігатися нестійкість повздовжніх хвиль. Виняток складає лише квадратичний спектр - в цьому випадку власні хвилі виявляються стаціонарними.