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Action of Random Force on Gunn Domain*

F.G.Bass
Department of Physics, Bar-Ilan University, 52 100 Ramat-Gan, Israel
R.Bakanas
Semiconductor Physics Institute, Gostauto 11, LT-2600, Vilnius, Lithuania

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The propagation of Gunn layers (GL), i.e. nonlinear waves of electron density distribution in a semiconductor specimen, under the influence of current fluctuations, which may be characterized as a white Gaussian noise, is considered. An iterative scheme of perturbation expansion is developed to deduce the statistical properties of GL's analyzed in the hydrodynamical approach. The average characteristics of GL's are examined for two statistical ensembles that describe essentially different physical situations. It is shown that the current fluctuations significantly influence the shape of the averaged GL field, the averaged velocity of GL and the voltage drop over the specimen are not influenced by the current fluctuations in the both cases considered. The diffusion equation which describes the spreading of the averaged profile of GL's is derived for the particular ensemble of GL's with randomly distributed phases. Some numerical estimations are presented.

1. Introduction. Influence of small perturbations on nonlinear wave

The propagation of nonlinear excitations under the influence of regular as well as random perturbations has been intensively investigated theoretically in the last few years, mainly in the case of conservative systems [1-3]. The nonlinear excitations in essentially dissipative systems, which could be described by nonlinear diffusion equations, are widely known in various fields of physics, e.g. superconductivity, solid state physics, plasma physics, biophysics, etc.[4]. The analytical description of such excitations, spatially localized waves, under influence of temporal as well as spatial perturbations is rather complicated even in the presence of small perturbations. In the most simple situations the evolution of such excitations under influence of small perturbations may be described by diffusion equation of the following type:

$$\frac{\partial u}{\partial \tau}+\hat{\Lambda}u=\beta f(y,\tau), \qquad (1.1)$$

where $\hat{\Lambda}$ is the nonlinear operator,

$$\hat{\Lambda}=-d\frac{\partial^2}{\partial y^2}+p(u)\frac{\partial}{\partial y}+r(u), \qquad (1.2)$$

and the torque $f(y,\tau)$ describes a small $(\beta<<1)$ perturbation. Here, τ is time, y denotes the moving coordinate, $y=x-c_0\tau$, both measured in the dimensionless units, and c_0 is the velocity of free (unperturbed) wave. Thus, Eqs. (1.1) and (1.2) are written in the moving coordinates. Taking the explicit expressions of $p(u)$, $r(u)$ and allowing $\beta=0$ in Eq. (1.1) one may obtain the unperturbed solutions $u_0(y)$ that describe free waves. To obtain the solution $u_0(y,\tau)$ of Eq. (1.1) in the presence of small disturbing torque f we apply a perturbation scheme that is based on the properties of the linear operator $\hat{L}$, intimately related to the translational mode $\overline{Y}(y)\propto du_0(y)/dy$,

$$\hat{L}\overline{Y}=0,\quad \hat{L}=-d\frac{d^2}{dy^2}+Q(y)\frac{d}{dy}+U(y),$$

$$Q(y)=p[u_0(y)],\quad U(y)=p'(y)\frac{du_0(y)}{dy}+r(y), \qquad (1.3)$$

where the prime denotes the derivative with respect to u_0, i.e. $p'(y)=dp[u_0(y)]/du_0$, etc. Here and in following, the overline denotes the quantities closely related to the translational mode. The existence of translational mode evidently follows from translational invariance of the operator $\hat{\Lambda}$. We assume that

$$Q(y)\to 0 \text{ if } y\to\pm\infty. \qquad (1.4)$$

Thus, the eigenfunctions Y_α of operator $\hat{L}$ make up a complete set and the corresponding eigenvalues λ_α are real.

$$\hat{L}Y_\alpha=\lambda_\alpha Y_\alpha. \qquad (1.5)$$

Owing to the fact that operator $\hat{L}$ is nonhermitian it is possible to express the functions Y_α through the

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* *The short communication about this problem was communicated in [11]*

eigenfunctions X_α of the hermitian operator $\hat{K} = S^{-1}(y)\hat{L}S(y)$ [5,6],

$$Y_\alpha(y) = S(y)X_\alpha(y),$$
$$S(y) = \exp\left\{(2d)^{-1}\int_{-\infty}^{y} dxQ(x)\right\}. \qquad (1.6)$$

In the presence of the small disturbing torque the solution of Eq. (1.1) $u(y,\tau)$ may be presented in the following way:

$$u(y,\tau) = u_0[y+s(\tau)]+\Delta u(y,\tau), \ \ \Delta u \ll u_0 \qquad (1.7)$$

where the parameter $s(\tau)$ denotes the phase shift and $\Delta u(y,\tau)$ describes the distortion of the wave due to presence of the perturbation f. Substituting (1.7) into (1.1) one rewrites Eq. (1.1) as follows:

$$\frac{du_0(\xi)}{d\xi}\frac{ds}{d\tau}+\frac{\partial \Delta u(\xi,\tau)}{\partial\tau}+\hat{L}\Delta u(\xi,\tau)=F(\Delta u,s;u_0;f),$$
$$F(\xi,\tau)=\beta f(\xi,\tau)-g\left[\Delta u(\xi,\tau),s(\tau);u_0(\xi)\right], \qquad (1.8)$$

where $\xi = y+s(\tau)$ is the comoving coordinate, i.e. the coordinate moving with the perturbed wave. The additional "torque" g includes the nonlinear term with respect to the small quantities $ds/d\tau$ and Δu,

$$g=\left[p(u_0+\Delta u)-p(u_0)\right]\left(\frac{du_0}{d\xi}+\frac{\partial\Delta u}{\partial\xi}\right)+$$
$$+r(u_0+\Delta u)-r(u_0)+\frac{ds}{d\tau}\frac{\partial\Delta u}{\partial\xi}- \qquad (1.9)$$
$$-\left[p'(u_0)\frac{du_0}{d\xi}+r'(u_0)\right]\Delta\tau,$$

In deriving (1.8) from (1.1) the relationship $\hat{\Lambda}u_0(\xi)=0$ has been taken into account. In the case of small perturbation we expand the quantities s, Δu and F in a power series of the small parameter β,

$$\phi(\xi,\tau)=\sum_{n=1}\beta^n\phi^{(n)}(\xi,\tau), \ \ \phi=s,\Delta u, F. \qquad (1.10)$$

By substitution of Eq. (1.10) into (1.8) the following evolution equation may be obtained in the β^n order approximation:

$$\frac{du_0(\xi)}{d\xi}\frac{ds^{(n)}}{d\tau}+\frac{\partial\Delta u^{(n)}(\xi,\tau)}{\partial\tau}+$$
$$+\hat{L}\Delta u^{(n)}(\xi,\tau)=F^{(n)}(\xi,\tau). \qquad (1.11)$$

Expanding the functions $\Delta u^{(n)}$ and $F^{(n)}$ through the complete set of the eigenfunctions Y_α,

$$\Delta u^{(n)}(\xi,\tau)=\sum_\alpha T_\alpha^{(n)}(\tau)Y_\alpha(\xi),$$
$$F^{(n)}(\xi,\tau)=\sum_\alpha F_\alpha^{(n)}(\tau)Y_\alpha(\xi), \qquad (1.12)$$

one rewrites the Eq. (1.11) into "L - representation"

$$\frac{dT_\alpha^{(n)}}{d\tau}+\left\langle Y_\alpha\left|\frac{du_0}{d\xi}\right.\right\rangle\frac{ds^{(n)}}{d\tau}+\lambda_\alpha T_\alpha^{(n)}=F_\alpha^{(n)}(\tau),$$
$$F_\alpha^{(n)}(\tau)=\left\langle Y(x)\left|F^{(n)}(x,\tau)\right.\right\rangle. \qquad (1.13)$$

Here, the following notation has been introduced

$$\langle Y_\alpha|\psi\rangle=\int_{-\infty}^{\infty}dx\,Y_\alpha^+(x)\psi(x,\tau), \qquad (1.14)$$

where the dagger indicates the eigenfunction of the adjoint operator $\hat{L}^+ = S^{-1}\hat{K}S$,

$$Y_\alpha^+(x)=S^{-1}(x)X_\alpha^*(x). \qquad (1.15)$$

Now, the evolution equations that describe the needed functions, $S^{(n)}(\tau)$ and $T_\alpha^{(n)}$, may be simply obtained from Eq. (1.13), if one takes into account that the contribution of translational mode $\overline{Y}(y)$ may be fully included into phase shift $S^{(n)}(\tau)$. Thus, substituting $d\overline{T}^{(n)}/d\tau=0$ into (1.13), one gets immediately

$$s^{(n)}(\tau)=\left\langle\overline{Y}\left|\frac{du_0}{d\xi}\right.\right\rangle^{-1}\int_0^\tau dt\overline{F}^{(n)}(t),$$
$$T_\alpha^{(n)}(\tau)=\int_0^\tau dtF_\alpha^{(n)}(\tau)\exp\left[\lambda_\alpha(t-\tau)\right]. \qquad (1.16)$$

In deriving (1.16) from (1.13) we have supposed that the perturbing torque $f(y,\tau)$ has been "turned on" at the initial moment τ=0.

The Eqs. (1.16) used in conjunction with (1.7), (1.10) and (1.12) allow to describe the evolution of nonlinear wave under small perturbations with the needed accuracy. Additionally, with the help of (1.16) the stochastic characteristics of the wave may be also deduced in the case of random torque $f(y,\tau)$.

The described perturbation scheme has been applied to a particular case of Gunn wave (GW) influenced by random, spatially homogeneous perturbations, which, as assumed, originate from the current fluctuations in external circuit of the specimen.

2. Description of free wave. Characteristics of random perturbations

In the case of semiconductor sample with N-type current-voltage characteristics Eq. (1.1) describes the evolution of the electric field $\mathcal{E}(y,\tau)$ distribution along the specimen [5-7], i.e. $u(y,\tau)=\mathcal{E}(y,\tau)$ in this case. The disturbing torque f may be induced by inhomogeneities of the doping profile in a sample or originates due to the current or voltage deviations in the external circuit, in the case of GW's. It is well known [5-7] that the functions $p(\mathcal{E})$ and $r(\mathcal{E})$ in (1.2) are simply related to the current-voltage characteristic $w(\mathcal{E})$ of the specimen,

$$p(\mathcal{E})=\lambda_G^{-1}\left[w(\mathcal{E})-c_0\right],\ r(\mathcal{E})=w(\mathcal{E})-J_0, \tag{2.1}$$

where J_0 denotes the total current in the specimen in the case of free GW, and the parameter λ_G introduced for convenience is defined below.

The unperturbed solution $\mathcal{E}_0(y)$ of Eq. (1.1) describes the free Gunn waves, i.e. nonlinear excitation of two types, Gunn layers (accumulation and depletion waves) and Gunn domains [7]. For description of GW's by Eqs. (1.1), (1.2) and (2.1) we use the dimensionless units: time $\tau=t/\tau_M$ is measured in the units of dielectric relaxation time τ_M and the moving coordinate $y=x/l_G-c_0\tau$ is scaled in the units of characteristic length l_G that characterizes the electric field distribution over specimen in the free GW case. The quantity l_G indicates the size of GW nucleus, i.e. it shows the characteristic distance of charge density localization, $\rho(x)\sim d\mathcal{E}_0(x,t)/dx$, in the free GW. The following dimensionless quantities are also introduced into Eqs. (1.1), (1.2) and (2.1): $\mathcal{E}=\mathcal{E}/\mathcal{E}_p$, $w=v/v_p$, $d=\left(l_D/l_G\right)^2$, $l_D=\left(D\tau_M\right)^{1/2}$, $\lambda_G=l_G/l_V$, $l_V=\tau_M v_p$ where $\mathcal{E}$ describes the electric field in GW, the parameters $\mathcal{E}_p$, v_p correspond to the electric field and the electron drift velocity at the peak of current-voltage characteristics, and D is the diffusion coefficient.

To evaluate the average characteristics of GW under random perturbation $f(\tau,y)$ one has to define both the dependence $w=w(\mathcal{E})$ and the stochastic properties of perturbation $f(\tau,y)$ explicitly. In addition, the explicit expressions of eigenfunctions $Y_\alpha(\xi)$ and corresponding eigenvalues λ_α are needed, and the solution of unperturbed GW $\mathcal{E}_0(y)$ is also required, as is seen from Eqs. (1.7), (1.16) and (1.12).

Here, we consider the average characteristics of Gunn layers (GL) influenced by the current fluctuations which originate, as assumed, in the external circuit of the sample. The disturbing torque f is closely related to the density of total current deviations $\delta J(\tau)$ in this case [5],

$$f=\delta J(\tau)=J(\tau)-J_0 \tag{2.2}$$

A simple theoretical model allowing the analytical description of free GL's may be obtained with the current-voltage characteristics $w=w(\mathcal{E})$ taken in the following form [5]:

$$w(\mathcal{E})=a+b\sin\left[\gamma(\mathcal{E}-F)\right], \tag{2.3}$$

where a, b, γ, F are free parameters. The relation (2.3) describes the needed dependence $w(\mathcal{E})$ in the electric field region defined by $0\le\gamma(\mathcal{E}-F)\le 2\pi$. The shape of the w-$\mathcal{E}$ characteristic outside this region is nonessential for our purposes if one takes $J_0=a$. The "equal areas rule" is satisfied in this case, and the parameters of free GL are fully determined by the $w(\mathcal{E})$ dependence in the "inside region" [5]. Now, the solution that describes the free GL may be obtained from (1.1) by substitution of Eqs. (2.1) and (2.3) into (1.1). Obviously, the right side of (1.1) has to be neglected ($\beta=0$). In the strong diffusion case ($l_D>>l_V$) one gets [5]

$$\mathcal{E}_0(y)\approx\overline{F}+2\gamma^{-1}\arctan\sinh(\pm y), \tag{2.4}$$

where $\overline{F}=F+\Delta/2$ and $\Delta=2\pi/\gamma=\left(\mathcal{E}_M-\mathcal{E}_m\right)$ denotes the increment of electric field in GL wave. The quantities $\mathcal{E}_M$ and $\mathcal{E}_m$ indicate the extreme values of electric field in GL. The sign + or − corresponds to the accumulation and depletion wave, respectively. The function $\psi(y)=\arctan\sinh(\pm y)$ in (2.4) is defined as, $-\frac{\pi}{2}\le\psi(y)\le\frac{\pi}{2}$. Note, the characteristic distance describing the charge density distribution in GL wave, $\rho(x) \sim d\mathcal{E}_0(x)/dx$, i.e. the extent of GL nucleus in the x-coordinates is of an order of $l_G = l_V \lambda_G$ where $\lambda_G = (\overline{w}')^{-1/2} l_D / l_V >> 1$. Now, substituting (2.1), (2.3) and (2.4) into (1.3) one may obtain the required functions Y_α and the corresponding eigenvalues λ_α explicitly [5]

$$\overline{X}(\xi) = 2^{-1/2}\mathrm{sech}\xi,\ \overline{\lambda} = 0,$$

$$X_k(\xi) = \left[2\pi(k^2+1)\right]^{-1/2}(\tanh\xi - ik)\exp(ik\xi),$$

$$\lambda(k) = \lambda_a(k^2+1)$$

$$S(\xi) = \exp\left[\pm(\gamma\lambda_G)^{-1}\mathrm{sech}\xi\right] \approx 1, \tag{2.5}$$

where parameter $\lambda_a = b\gamma = \overline{w}'$ indicates the bottom of continuous spectrum. Note, that the eigenvalue $\overline{\lambda}$ which corresponds to the translational mode is a single discrete eigenlevel in the considered case. Hence, the additional field $\Delta\mathcal{E}(\xi,\tau)$ is fully determined by contribution of eigenfunctions that correspond to the continuous spectrum, as one can see from Eqs. (1.12) and (1.16).

To evaluate the average characteristics of GL let us define the statistical properties of the disturbing torque $\delta J(\tau)$ more exactly. It is supposed in the following that the current oscillations $\delta J(\tau)$ correspond to the white Gaussian noise and are "turned on" at $\tau = 0$, as assumed in the derivation of Eqs. (1.16). Thus, the statistical properties of $\delta J(\tau)$ are characterized as follows

$$\langle \delta J(\tau)\rangle = 0,$$

$$\langle \delta J(\tau_1)\delta J(\tau_2)\ldots\delta J(\tau_n)\rangle = \delta_{n/2,r}\sum B_{ij}\ldots B_{ml},$$

$$B_{ij} = \langle \delta J(\tau_i)\delta J(\tau_j)\rangle = 2\sigma^2\delta(\tau_i - \tau_j), \tag{2.6}$$

where r is an integer, Σ indicates the sum of correlators B_{ij} covering all pair combinations of indices. The parameter σ in (2.6) characterizes the external noise intensity, and the bracket $\langle\ \rangle$ denotes the ensemble average, i.e. the average over all the possible realizations of $J(\tau)$.

Finally, let us specify the statistical ensembles of GL's more exactly, i.e. let us describe the averaging procedure more accurately. It is well known [1,2] that in the considerations of stochastic properties of solitary waves in the conservative systems two different procedures of the averaging are common. They correspond to the averages over two separate statistical ensembles, both describing a "random walk" of solitary waves.

The first one, introduced by Kaup (see[1]), incorporates the averaging "relative to the moving wave" and seeks to describe the statistical characteristics of the waves that have "arrived" to a certain point of the specimen. The corresponding ensemble of GL's includes the waves of a randomly distorted shapes only. The averaging over irregular shifts of the phases $s(\tau)$ is ignored in this case. This ensemble will be called the "small ensemble". Thus, the averages over the small ensemble describe the mean characteristics of the waves that arrived to some point of the specimen, where by "coordinate of the wave" we mean the position of nucleus center of the wave.

The second averaging proposed by Wadati [7] involves all irregular parameters of the randomly disturbed wave. It corresponds to the averages over "complete ensemble" of the "random waves" and includes the random distribution of separate waves not only over the distorted shapes but also over the available phase displacements. Each member of the ensemble, a single solitary wave, is characterized not only by its individual shape but also by the unique location, both being randomly distributed in space. Thus, the complete ensemble takes into account all irregular parameters of the wave. Obviously, average over the complete ensemble describes the essentially different physical situation. The complete ensemble corresponds to the infinite collection of the macroscopically identical specimens characterized by unique realizations of the random torque. Hence, the averages over complete ensemble may be attributed to the mean characteristics of GL's distributed over the macroscopically identical specimens.

To describe the statistical properties more extensively we analyze the average characteristics of GL's in the both ensembles. By the method of perturbation expansion described above we examine the evolution of averaged GL field $\langle \mathcal{E}(x,t)\rangle$ in the presence of random current oscillations $J(\tau)$ that, as assumed, are of Gaussian type.

3. Averages over small ensemble

By averaging over the small ensemble we intend to evaluate the average characteristics of GL in the "comoving coordinates" ξ, i.e. in the coordinates moving with the nucleus center of the perturbed wave. Thus, from (1.7) one has

$$\overline{\mathcal{E}} = \mathcal{E}_0(x,t) + \overline{D\mathcal{E}}(x,\tau), \tag{3.1}$$

where overline denotes the averaged field, $\overline{D\mathcal{E}}(x,t)=\langle D\mathcal{E}(x,t)\rangle$. From (1.16), in the conjunction with Eqs. (2.2), (1.13), (1.14) and (2.6), one gets that $\langle T_k^{(1)}\rangle = 0$. Hence, from $\overline{D\mathcal{E}}^{(1)}(x,\tau)=0$, and the shape of the averaged field distribution along the specimen $\overline{\mathcal{E}}_1(x)=\langle \mathcal{E}_0(x)+D\mathcal{E}^{(1)}(x,t)\rangle$ strictly follows that of the free wave, if the lowest approximation of perturbation expansion (1.10) is used. Now it may be concluded that the realizations of the disturbed GL field are symmetrically distributed around the field $\mathcal{E}_0(\xi)$ that describes the free GL. Indeed, after evaluation of the probability distribution $F_T(T_k)=\langle\delta(T_k-T_k^{(1)}(\tau))\rangle$ of the parameter T_k, with the help of (1,16), (2.2) and (2.6), one gets

$$F_T(T_k)=\left[4\pi\sigma_T^2(\tau)\right]^{-\frac{1}{2}}\exp\left[-T_k^2/4\sigma_T^2(\tau)\right], \quad (3.2)$$

where

$$\sigma_T^2(\tau)=\lambda_k^{-1}\langle Y_k|1\rangle^2\sigma^2\left[1-\exp(-2\lambda_k(\tau)\right].$$

Thus, the obtained distribution $F_T(T_k)$ is Gaussian and its dispersion $\sigma_T^2(\tau)$ approaches the fixed value $s_T^2=\lambda_k^{-1}\langle Y_k|1\rangle^2\sigma^2$ in the long time limit $\tau >> \tau_R$. Here $\tau_R=\lambda_a^{-1}=1/\overline{w}'$ indicates the dielectric relaxation time. Moreover, from (1.16) used in conjunction with (2.2) and (2.6) follows that $\langle s^{(1)}(\tau)\rangle = 0$. Hence, the average comoving coordinate $\langle\xi_1\rangle = y$ obeys the nucleus center of the free GL. Taking into account that the first order field $\overline{\Delta\mathcal{E}}^{(1)}(\xi,\tau)$ vanishes, let us evaluate the second-order average $\overline{\Delta\mathcal{E}}^{(2)}(\xi,\tau)$. It is seen from Eqs. (1.12) and (1.16) that one needs to evaluate the averaged torque $\overline{f^{(2)}}=\langle f^{(2)}(\xi,\tau)\rangle$, to obtain the mean field $\overline{\Delta\mathcal{E}}^{(2)}(\xi,\tau)$. In the case of symmetric GL, $\lambda_G>>1$, from Eq. (1.9) used in conjunction with (2.1), follows

$$\overline{f^{(2)}}\approx-\left\{\left\langle\frac{ds^{(1)}}{dt}\frac{\partial\Delta\mathcal{E}^{(1)}}{\partial\xi}\right\rangle+2^{-1}w''\left[\mathcal{E}_0(\xi)\right]\left\langle\Delta\left(\mathcal{E}^{(1)}\right)^2\right\rangle\right\}, \quad (3.3)$$

A small term, proportional to λ_G^{-1}, has been neglected in the derivation of (3.3). In seeking the field $\overline{\Delta\mathcal{E}}^{(2)}(\xi,\tau)$ we are interested in the "steady state" which develops at sufficiently long times after the random torque $\delta J(\tau)$ has been turned on. Thus, we do not analyze the relaxation process to the "steady state" that takes place at the initial moment. Performing the averaging in (3.3) one gets with the help of (1.12) and (1.16) (Appendix A)

$$\overline{\Delta\mathcal{E}}^{(2)}(\xi)=\int_{-\infty}^{\infty}dk\lambda_k^{-1}\langle Y_k|B\rangle Y_k(\xi)= \int_{-\infty}^{\infty}dzB(z)G_1(z,\xi), \quad (3.4)$$

where

$$G_1(z,\xi)=\int_{-\infty}^{\infty}dk\lambda_k^{-1}Y_k^+(z)Y_k(\xi),$$

$$B(z)=2^{-1}\sigma^2\gamma^2\left\langle\overline{Y}\left|\frac{d\mathcal{E}_0}{dx}\right.\right\rangle\times \left\{\langle\overline{Y}|1\rangle^2+\left[1-\langle\overline{Y}|1\rangle\overline{Y}(z)\right]^2\right\}\frac{d\overline{Y}(z)}{dz}. \quad (3.5)$$

It is seen from (3.4) and (3.5) that the additional fields $\overline{\Delta\mathcal{E}}^{(2)}(\xi)$ does not depend on time, i.e. it describes the "steady state" which becomes settled at the times exceeding the dielectric relaxation time ($\tau>>\tau_R$) (see Appendix A). Owing to the fact that the signature of $\frac{d\mathcal{E}_0(y)}{dy}$ is opposite for accumulation or depletion wave case, one may conclude from (3.4) that the additional field, $\overline{\Delta\mathcal{E}}^{(2)}(\xi)$ are also strictly opposite for these two cases. Performing the needed integrations in (3.4) by use of the closure condition (A.7), one gets (Appendix B)

$$\overline{\Delta\mathcal{E}}^{(2)}(\xi)\approx\lambda_a^{-1}\left\{B(\xi)-\langle\overline{Y}|B\rangle\overline{Y}(\xi)\right\} \quad (3.6)$$

Evidently, the expression (3.6) is in a good agreement with previous our assumption: the additional field $\overline{\Delta\mathcal{E}}^{(2)}(\xi)$ does not include the contribution of the translational mode. Taking the explicit expressions (2.5) and (3.5), we finally obtain from (3.6)

$$\overline{\Delta\mathcal{E}}^{(2)}(\xi)\approx\mp(8\lambda_a)^{-1}\gamma\sigma^2\times$$
$$\times\left\{2\pi^2+(2-\pi\cosh^{-1}\xi)^2\right\}\frac{\tanh\xi}{\cosh\xi}. \qquad (3.7)$$

The results (3.6) and (3.7) are approximate ones. Starting from (3.4) one can obtain the needed dependence $\overline{\Delta\mathcal{E}}^{(2)}(\xi)$ more accurately. Indeed, the exact evaluation of the "response function" $G_1(z,\xi)$ is possible by use of the method of contour integration in the complex k-plane (Appendix C)

$$G_1(z,x)\approx\left(8\lambda_a\cosh z\cosh x\right)^{-1}\times$$
$$\times\left\{\exp(-2z_>)+\exp(-2z_<)-2|z-x|\right\} \qquad (3.8)$$

where $z_>=\max\{z,x\}$ and $z_<=\min\{z,x\}$. Now the required field $\overline{\Delta\mathcal{E}}^{(2)}(\xi)$ may be evaluated by substitution of Eqs. (3.5) and (3.8) into (3.4). However. the needed integration in (3.4) cant not be carried out rigorously by analytical methods. Thus, the obtained expression (3.8) is useful for the numerical evaluation of the field $\overline{\Delta\mathcal{E}}^{(2)}(\xi)$. The performed numerical simulations show that the additional field $\overline{\Delta\mathcal{E}}^{(2)}(\xi)$ given by (3.6) is in a good agreement with the strict result, obtained with the help of (3.8), within accuracy of few percents.

Finally, we present the explicit result which describes the averaged GL field in the second order approximation. From Eq. (3.1) used in conjunction with (2.4) and (3.7), it follows

$$\overline{\mathcal{E}}_2(\xi)=\mathcal{E}_0(\xi)+\overline{\Delta\mathcal{E}}^{(2)}(\xi)\approx\mathcal{E}_0\left(\frac{\xi}{L_G(\xi)}\right),$$
$$L_G(\xi)=1+\lambda_a^{-1}(\sigma\gamma/4)^2\left[2\pi^2+(2-\pi\cosh^{-1}\xi)^2\right]\times$$
$$\times\xi^{-1}\tanh\xi. \qquad (3.9)$$

It is seen from (3.9) that the averaged field $\overline{\mathcal{E}}(\xi)$ is similar to that which describes the free GL with a slightly distorted nucleus. One can see that "deformation" of the free wave induced by random perturbations $\delta J(\tau)$ are inhomogeneous and more strongly pronounced in the region of nucleus center of the free GL. The distortion of the nucleus $\Delta L_G(\xi)=$ $=L_G(\xi)-1$ is proportional to the noise intensity σ^2 and becomes negligible in the region outside the GL nucleus, i.e. $\Delta L_G(\xi)\to 0$ when $\xi\to\pm\infty$. Thus, the averaged field distribution along the specimen $\overline{\mathcal{E}}(\xi)$ in the outside region $\xi>>1$ practically coincides with that describing the free GL.

It is interesting to note that the average comoving coordinate $\langle\xi_2\rangle=y+\langle s^{(2)}(\tau)\rangle$, which has been evaluated with the help of (1.16), strictly follows the nucleus center of free GL, i.e. $\langle\xi_2\rangle=y$.

4. Complete ensemble averages. General relations

In evaluating the averages over the "complete ensemble" one has in mind that the position of GL are also randomly distributed along a specimen. By "position of GL", as in above, we mean the nucleus center of GL. Accordingly to (1.7) the average GL field is defined now as follows

$$\overline{\mathcal{E}}=\overline{\mathcal{E}}_0(y,\tau)+\overline{\Delta\mathcal{E}}(y,\tau), \qquad (4.1)$$

where overline indicates the GL field averaged over the complete ensemble, $\overline{\mathcal{E}_0}(y,\tau)=<<\mathcal{E}_0(\xi)>>$, etc., where sign $<<\,>>$ denotes averaging over the complete ensemble. In evaluating of the "complete averages" we will restrict ourselves by the first order approximation. To simplify denotations, the indexes will be omitted. The quantities s and $\Delta\mathcal{E}$ now correspond to those obtained in the first order approximation, i.e. $s=s^{(1)}$ and $\Delta\mathcal{E}=\Delta\mathcal{E}^{(1)}$.

Let us evaluate the mean field $\overline{\mathcal{E}}(y,\tau)$. By use of translational operator $\hat{T}(s)=\exp(s\partial/\partial y)$ one gets from (1.7) with the help of Eqs. (1.16), (2.2) and (2.6) (Appendix D)

$$<<\mathcal{E}_0(\xi,\tau)>>=<<\hat{T}(s)\mathcal{E}_0(y)>>=\hat{d}(s)\mathcal{E}_0(y), \qquad (4.2)$$

where $\hat{d}(s)$ denotes the "diffusion operator"

$$\hat{d}(s)=\exp\left[2^{-1}<s^2(\tau)>\frac{\partial^2}{\partial y^2}\right]. \qquad (4.3)$$

From (4.2) immediately follows

$$\frac{\partial\overline{\mathcal{E}_0}}{\partial\tau}=\overline{D}\frac{\partial^2\overline{\mathcal{E}_0}}{\partial y^2},$$

$$\overline{D}=\frac{1}{2}\frac{d<<s^2(\tau)>>}{d\tau}=\left\{\sigma\langle\overline{Y}|1\rangle\left\langle\overline{Y}\left|\frac{d\mathcal{E}_0}{dy}\right.\right\rangle^{-1}\right\}^2, \qquad (4.4)$$

where the average $\langle\langle s^2(\tau)\rangle\rangle$ is evaluated with the help of Eqs. (1.16) and (2.6). The obtained relation shows the mean field $\overline{\mathcal{E}}_0$ obeys the diffusion equation characterized by diffusion coefficient $\overline{D}$ which is expressed in terms of the average magnitude of the phase displacements $s(\tau)$. Note, that characteristic time of diffusive spreading of GL $\tau_D = L_G^2/\overline{D}$ is of the order of $1/\overline{D}$ if one takes into account that the size of nucleus of the free GL L_G is of the order of unity. One can see that simple relationship (4.4) that describes the diffusive spreading induced by the "random walk" of GL phase $s(\tau)$ is a characteristic feature of Gaussian process. There are the conditions (2.6) which ensure the simple relation (4.3) between averaged translations of GL and "diffusion operator" $\hat{d}$ (see Appendix D).

The additional field $\overline{\Delta\mathcal{E}}(y,\tau)$, which results from the contribution of delocalized eigenfunctions $Y_k(\xi)$, may be evaluated from (1.12), (1.16) and (2.2) by the averaging procedure being performed in accordance with the relations (2.6),

$$\overline{\Delta\mathcal{E}}(y,\tau) = \int_{-\infty}^{\infty} dk \langle Y_k | 1\rangle \times \times \int_0^{\tau} dt \exp[\lambda_k(t-\tau)] \langle\langle \delta J(t)\hat{T}[s(\tau)]\rangle\rangle Y_k(y) \quad (4.5)$$

where the eigenfunction $Y_k(\xi)$, which appeared in (1.12), is introduced into (4.5) with the help of the translational operator, $Y_k(\xi) = \hat{T}(s)Y_k(y)$. The presence of the factor $\langle\langle \delta J\hat{T}(s)\rangle\rangle$ in integrand of (4.5) evidently shows that the additional field $\overline{\Delta\mathcal{E}}$ comes from correlation between the phase shifts of GL and the distortions of the shape of GL, both originating from random deviations of the current $\delta J(\tau)$. Performing the averaging in (4.5) with (2.6) one gets (Appendix E)

$$\overline{\Delta\mathcal{E}}(y,\tau) = \hat{d}(s;y)\frac{d}{dy}\int_{-\infty}^{\infty} dk K_k(\tau)\lambda_k^{-1} r_k(y), \quad (4.6)$$

where

$$r_k(y) = \langle Y_k | 1\rangle Y_k(y),$$

$$K_k(\tau) = 2\sigma^2 \langle \overline{Y} | 1\rangle \left\langle \overline{Y} \left| \frac{d\mathcal{E}_0(y)}{dy} \right.\right\rangle^{-1} [1-\exp(-\lambda_k\tau)].$$

It is seen that increase of the additional field $\overline{\Delta\mathcal{E}}$ at the initial moments after the current oscillations have been "turned on" is determined by the factor $K_k(\tau)$. The rise of $\overline{\Delta\mathcal{E}}$ is characterized by the dielectric relaxation time $\tau_R = \lambda_a^{-1}$, as follows from (4.6). At a sufficiently large time, exceeding the dielectric relaxation time, the magnitude of $K_k(\tau)$ approaches the fixed value, and the additional field $\overline{\Delta\mathcal{E}}(y,\tau)$ obeys the diffusion equation (4.4), as one can see from (4.6). Thus, the total GL field $\overline{\mathcal{E}}(y,\tau)$ at a sufficiently large times $\tau \gg \tau_R$ also obeys the diffusion equation

$$\frac{\partial\overline{\mathcal{E}}}{\partial b} = \frac{\partial^2\overline{\mathcal{E}}}{\partial y^2}, \quad (4.7)$$

where $b=\tau/\tau_D$ is the time duration measured in the units of characteristic "diffusion time" $\tau_D = 1/\overline{D}$. Consequently, the shape of the averaged GL field $\overline{\mathcal{E}}(y,\tau)$ is fully determined by diffusive spreading. It is interesting to note that similar diffusion equation which describes the averaged field of K-dV soliton under random perturbations has been deduced in [7].

Now one can see that the solution of Eq. (4.7) may be presented as a convolution of Gaussian function and averaged field distribution $\overline{\mathcal{E}}(y,b_0)$ given at any initial time $b=b_0$

$$\overline{\mathcal{E}}(y,b) = 2(\pi\Delta b)^{-1/2} \times \times \int_{-\infty}^{\infty} dx \exp[-(y-x)^2/4\Delta b]\overline{\mathcal{E}}(x,b_0), \quad (4.8)$$

where $\Delta b = b - b_0$. Integrating the both sides of (4.8) over y one concludes that the voltage drop over the specimen does not depend on b, i.e. it is not changed due to the presence of current oscillations $\delta J(\tau)$. Thus, one may expect that the mean "deformations" of GL, which have been induced by fluctuations of $\delta J(\tau)$, are distributed antisymmetrically with respect to the nucleus center of free GL. For our purposes it is useful to rewrite the Eq. (4.8) in the following manner:

$$\overline{\mathcal{E}}(y,b) = \int_{-\infty}^{\infty} dq \exp(-\Delta b q^2 + iqy)\overline{\mathcal{E}}(q,b_0), \quad (4.9)$$

where

$$\overline{\mathcal{E}}(q,b_0) = (2\pi)^{-1}\int_{-\infty}^{\infty} dy \exp(-iqy)\overline{\mathcal{E}}(y,b_0).$$

A more detailed analysis of the field $\overline{\mathcal{E}}(y,\tau)$, given below, is based on the relations (4.8) and (4.9) used in conjunction with the explicit expressions (2.4), (2.5) and the initial condition $\overline{\mathcal{E}}(y,\tau=0)=\mathcal{E}_0(y)$.

5. Complete ensemble averages. Evolution of the mean field

The explicit expressions of the averaged field $\overline{\mathcal{E}}(y,\tau)$ may be simply deduced from (4.8) and (4.9) in the limiting cases of short and long times, i.e. when $b<<1$ or $b>>1$.

We suppose in the following that the diffusive spreading of $\overline{\mathcal{E}}(y,\tau)$ is sufficiently slow as compared to the rate of dielectric relaxation process, i.e. we assume that $\tau_D>>\tau_R$. The characteristic time τ_D may be evaluated from Eq. (4.4) if the explicit expressions (2.4) and (2.5) have been taken into account: $\tau_D\cong(\Delta/\sigma)^2$. Hence, the diffusive spreading of the field $\overline{\mathcal{E}}(y,\tau)$ is slow if $\sigma^2<<\overline{w}'\Delta^2$. Now it is seen that the initial field $\overline{\Delta\mathcal{E}_0(y)}=\overline{\Delta\mathcal{E}}(y,\tau=\tau_0)$ may be evaluated from (4.6) if one assumes that $\tau_R.<<\tau_0<<\tau_D$. Neglecting the diffusive spread of the field $\overline{\Delta\mathcal{E}}(y,\tau=\tau_0)$ one can substitute $\hat{d}=1$ into (4.6), to obtain

$$\overline{\Delta\mathcal{E}_0(y)}\approx K_0\int_{-\infty}^{\infty}dk\lambda_k^{-1}\frac{dr_k(y)}{dy}\approx-\rho^2\frac{d^2\mathcal{E}_0}{dy^2}, \quad (5.1)$$

where

$$K_0=2\sigma^2\langle\overline{Y}|1\rangle\left\langle\overline{Y}\left|\frac{d\mathcal{E}_0(y)}{dy}\right.\right\rangle^{-1},$$

$$\rho^2=2\sigma^2\lambda_a^{-1}\left(\langle\overline{Y}|1\rangle\left\langle\overline{Y}\left|\frac{d\mathcal{E}_0}{dy}\right.\right\rangle^{-1}\right)^2=2\frac{D}{\lambda_a}=2\frac{\tau_R}{\tau_D}.$$

Deriving (5.1) we have assumed that $\lambda_k\tau_0\geq\tau_0/\tau_R>>1$. It is seen from (4.6) that the condition $\tau_0>>\tau_R$ implies that the "initial" time τ_0 is large enough and the dielectric relaxation processes, which develop at the initial moments after the random torque has been turned on, are almost over at the time τ_0.

The integration in (5.1) has been performed in a similar way as in (A.10) by taking into account that $B(z)=1$ (see Appendix A).

The condition $\tau_0<<\tau_D$ shows that the initial time τ_0 is small enough on a time scale which determines the diffusive spreading of the field $\overline{\mathcal{E}}(y,\tau)$. Thus, the averaged field $\overline{\mathcal{E}}(y,\tau)$ may be evaluated from (4.8) by use of the initial condition

$$\overline{\mathcal{E}}(y,b=0)\approx\overline{\mathcal{E}}(y,b_0=\tau_0\tau_D^{-1})\approx\mathcal{E}_0(y)+\overline{\Delta\mathcal{E}_0}(y). \quad (5.2)$$

It should be stressed that the condition (5.2) is valid if $b_R=\tau_R/\tau_D<<1$. In addition, it follows from the condition $\tau_0>>\tau_R$ that the relation (5.2) is useful for analysis of the averaged field at a sufficiently long times $b>>b_R$ exclusively. It is not valid in the short time region $b\leq b_R$, when the dielectric relaxation processes discussed above are still important. Hence, our analysis of the required field $\overline{\mathcal{E}}(y,\tau)$ is restricted by the condition $b>>b_R$. From Eqs. (4.8) and (5.2) used in conjunction with (5.1) one may conclude that the additional field $\overline{\Delta\mathcal{E}}(y,\tau)$ is proportional to the intensity of the "current noise" σ^2 and is small. It follows from (4.8) by use of Eqs. (5.2) and (5.1),

$$\overline{\mathcal{E}}(y,b)=\overline{\mathcal{E}}_0(y,b)-\rho^2\frac{\partial^2\overline{\mathcal{E}}_0(y,b)}{\partial y^2}. \quad (5.3)$$

Hence, to obtain the averaged field $\overline{\mathcal{E}}(y,b)$ the dependence $\overline{\mathcal{E}}_0(y,b)$ is required.

Let us evaluate $\overline{\mathcal{E}}_0(y,b)$ in the short time region $b_R<<b<<1$. Substituting $b_0=0$ into (4.8) one can see that the integrand $\exp[-(y-x)^2/4b]$ in (4.8) is a rapidly damped function of x, if $b<<1$. To perform integration in (4.8), one may expand the slowly varying function $\mathcal{E}_0(x)$ into the power series around the point $x=y$. Then, it is easy to get

$$\overline{\mathcal{E}}_0(y,b)=\mathcal{E}_0(y)+R_0(y,b),$$
$$R_0(y,b)=\sum_{m=1}^{\infty}(m!)^{-1}b^m\frac{d^{2m}\mathcal{E}_0(y)}{dy^{2m}}. \quad (5.4)$$

Retaining the leading term $m=1$ in sum (5.4) one gets from Eqs. (5.3), (5.4) and (2.4) approximately

$$\overline{\mathcal{E}}(y,b)\approx\mathcal{E}_0(y)+b\frac{d^2\mathcal{E}_0(y)}{dy^2}\approx\mathcal{E}_0\left[\frac{y}{L_G(y)}\right], \quad (5.5)$$

where $L_G=1+by^{-1}\tanh y$. Thus, the mean field $\overline{\mathcal{E}}(y,b)$ in the short time region is similar to that of free GL with a slightly smeared nucleus, the size of the nucleus being linearly increased in time. From (5.4) and (2.4) one can also see that $R_0(-y)=-R_0(y)$. Hence, the mean voltage drop over the specimen is not influenced by the random perturbations $\delta J(\tau)$.

In the long time region $b>>1$ the needed field $\overline{\mathcal{E}}_0(y,b)$ may be obtained from (4.9) by taking into account that the integrand $\exp(-bq^2)$ in (4.9) is a sharp function of q, if $b>>1$(Appendix F)

$$\overline{\mathcal{E}}_0(y,b)=\mathcal{E}_\infty(y,b)+R_\infty(y,b), \tag{5.6}$$

where

$$\mathcal{E}_\infty(y,b)=\overline{F}+\frac{\pi}{\gamma}\Phi\left(\pm\frac{y}{2\sqrt{b}}\right),$$
$$R_\infty(y,b)=\mp\frac{\pi}{\gamma}\sum_{n=0}^{\infty}\alpha_n Q_n(y,b). \tag{5.7}$$

Here

$$\Phi(x)=2\pi^{-1/2}\int_0^x dt\exp(-t^2) \tag{5.8}$$

is Frensel integral, and

$$Q_n(y,b)=y\frac{\partial^n}{\partial b^n}\left[b^{-3/2}\exp\left(-\frac{y^2}{4b}\right)\right]. \tag{5.9}$$

Coefficient α_n may be expressed in terms of Euler numbers E_n

$$\alpha_n=\left\{2\pi^{1/2}[2(n+1)]!\right\}^{-1}(2^{-1}\pi)^{2(n+1)}\mathcal{E}_{n+1}. \tag{5.10}$$

Retaining the leading term in (5.7) one gets from (5.3) and (5.7)

$$\overline{\Delta\mathcal{E}}(y,b)=-\rho^2\frac{\partial^2\overline{\mathcal{E}}_0(y,b)}{\partial y^2}\approx$$
$$\approx\pm\rho^2\pi^{1/2}(\gamma\, b)^{-1}z\exp(-z^2), \tag{5.11}$$

where $z=\left(2\sqrt{b}\right)^{-1}y$. Thus, the additional field $\overline{\Delta\mathcal{E}}$ diminishes in the infinite limit: $\overline{\Delta\mathcal{E}}(y,b)\sim\tau^{-3/2}$ if $\tau\to\infty$. Note, that the supplemental part of the field $R_\infty(y,b)$ is also decreasing as $\tau^{-3/2}$ in the long time limit.

From Eqs. (5.6) - (5.10) and (5.3) one can see that the averaged field $\overline{\mathcal{E}}(y,b)$ describes the localized wave which propagates with the unperturbed velocity c_0 and is appreciably smeared in comparison with the free GL. It is also seen from (5.7) and (5.9) that the following condition is satisfied: $R_\infty(-y)=-R_\infty(y)$. Therefore, neither supplemental (R_∞) nor additional ($\overline{\Delta\mathcal{E}}$) parts of the field influence the total voltage drop over the specimen. Neglecting the additional and supplemental parts of the field one has in asymptotically long time limit

$$\overline{\mathcal{E}}(y,b)\cong\mathcal{E}_\infty(y,b)=\overline{F}+\frac{\pi}{\gamma}\Phi\left(\pm\frac{y}{2\sqrt{b}}\right). \tag{5.12}$$

Hence, the mean field $\mathcal{E}_\infty(y,b)$ describes the kink-like wave characterized by a diffusively enlarged nucleus which is extended over the distances $|y|\le b^{1/2}$. The extreme values of the field in the "averaged wave" coincide with those of free GL, as seen from (5.12). This conclusion is in reasonable agreement with the earlier results concerning the dynamics of propagation of the single GL [5,7]. Indeed, by taking into account that the distortion of a single wave induced by small current deviation δJ is almost homogeneous in the region outside the nucleus of GL and is proportional to the magnitude of δJ, one can see that the extreme values of the field in the "averaged GL" will coincide with that of the free GL.

The smearing of the nucleus of the averaged wave may be interpreted as being appeared due to the random displacements of separate GL's along the specimen. Evaluating the probability distribution $F_s(s)=\left\langle\delta\left(s-s^{(1)}(\tau)\right)\right\rangle$ of the parameter s which describes the phase shift of GL, one gets by use of Eqs. (1.16), (2.2) and (2.6)

$$F_s(s)=\left[4\pi\sigma_s^2(\tau)\right]^{-1/2}\exp\left\{-\frac{s^2}{4\sigma_s^2(\tau)}\right\}, \tag{5.13}$$

where $\sigma_s^2(\tau)=\overline{D}\tau$ and the coefficient $\overline{D}$ is defined in (4.4). Thus, the nucleus centers of separate waves influenced by the random perturbation $\delta J(\tau)$ are randomly distributed along the specimen, the dispersion $\sigma_s^2(\tau)$ of the distribution $F_s(s)$ being linearly increased in time. It is interesting to note that the diffusive spread of K-dV solitons analyzed in [8,9] is of the same physical origin as described here.

6. Conclusions

Summarizing we conclude that the current fluctuations in a specimen influence the propagation of GL waves in two ways, by disturbing the GL phase and by transforming the shape of GL. In the case of Gaussian fluctuations the average characteristics of GL are found to be similar to those obtained in considerations of the "random walk" of solitary waves [1].

The "small ensemble" averages which describe the mean characteristics of GL being arrived to a certain point of the specimen show that the shape of averaged GL is slightly deviated from that describing the free GL. The deviations are proportional to the intensity of the "current noise" and are mainly localized in the nucleus region of free GL. They are antisymmetrically distributed with respect to the nucleus center of free GL, thus, the voltage drop in the averaged GL coincides that in the free GL.

The averages over "complete ensemble" show the infinite diffusive spreading of the averaged wave. At the times that significantly exceed the dielectric relaxation time after the current fluctuations have been "turned on", the averaged field of GL's obeys the diffusion equation, with the diffusion coefficient linearly increased with the intensity of the current noise. The average characteristics of GL's described by the complete ensemble are similar to those earlier obtained in considerations of the "random walk" of K-dV solitons [8,9].

We note that for the both ensembles considered here the mean velocity of GL as well as the extreme values of the field in averaged GL coincide with those of free GL. This conclusion is in obvious disagreement with the analogous result describing the "random walk" of the damped sine-Gordon kinks [1]. The "conservation laws" of the mean velocity c_0 and the averaged magnitudes of the wave, $\mathcal{E}_M$ and $\mathcal{E}_m$, that appear in the case of GL's, may be interpreted as being conditioned by the regime of the external circuit, $<J(\tau)>=J_0$. Note, that the regular current deviation δJ influences both, the magnitudes $\mathcal{E}_M$, $\mathcal{E}_m$, and the velocity c_0 of a single GL [5,7]. Thus, for instance, the total transit time of the averaged GL changes if current oscillations are not strictly random but contain also a regular part.

Finally, let us present some numerical estimations concerning the random walk of GL. The diffusive spread of averaged GL described by Eq. (4.7) is significant if the transit time of GL $\tau_T = L/c_0$ (L is the length of the specimen) considerably exceeds the characteristic "diffusion time" $\tau_D \cong (\Delta/\sigma)^2$. Thus, the spreading of GL wave is appreciable if intensity of random current oscillations σ is sufficiently high, $\sigma^2 \geq \sigma_T^2 = \Delta^2/\tau_T$. Taking the values $L=10^{-3}m$, $\tau_M=10^{-12}s$, $\Delta=1$ one gets in the standard units $\sigma_T^2 = 10^{-4} J_p^2 \tau_M \approx 10^{-2}\left(A/m^2\right)^2 s$ if $J_p = \rho_0 v_p \cong 10^7\ A/m^2$ and $\tau_T \cong 10^{-8}s$ (ρ_0 is the electric charge density of positive background in the specimen). Such spectral density of current noise may be achieved by the use of dynamical noise generators [10].

Let us evaluate the additional field $\overline{\Delta\mathcal{E}}^{(2)}(\xi)$ described by Eq. (3.7). Taking $\mathcal{E}_m \cong \Delta$ one can see from (2.4) and (3.7) that $\overline{\Delta\mathcal{E}}^{(2)}/\mathcal{E}_0 = (\sigma/\sigma_R)^2$, where $\sigma_R^2 = \Delta^2/\tau_R$. Taking $\tau_R/\tau_T \cong 10^{-4}$, one concludes that the additional field $\overline{\Delta\mathcal{E}}^{(2)}(\xi)$ is observable if $\sigma \geq 10^{-3}\sigma_T$. Such spectral density may be achieved by IMPAAT diodes.

Appendix

A. Performing the averaging in Eq. (3.3) by use of (2.6) one gets

$$\overline{f^{(2)}}(\xi,\tau) = -\left[A_1(\xi) + A_2(\xi,\tau)\right], \tag{A.1}$$

where

$$A_1(\xi) = \sigma^2 \langle \overline{Y} | 1 \rangle \left\langle \overline{Y} \left| \frac{d\mathcal{E}_0}{dz} \right.\right\rangle^{-1} \frac{dR(\xi)}{d\xi}, \tag{A.2}$$

$$A_2(\xi,\tau) = \sigma^2 w''\left[\mathcal{E}_0(\xi)\right] P(\xi,\tau). \tag{A.3}$$

The functions $R(\xi)$ and $P(\xi,\tau)$ are defined as follows:

$$R(\xi) = \int_{-\infty}^{\infty} dq\, r_q(\xi) = \int_{-\infty}^{\infty} dz\, G_0(z,\xi), \tag{A.4}$$

$$P(\xi,\tau) = \int_{-\infty}^{\infty} dq \int_{-\infty}^{\infty} dp (\lambda_q + \lambda_p)^{-1} r_q(\xi) r_p(\xi) a_{qp}(\tau), \tag{A.5}$$

where

$$a_{qp}(\tau) = \left\{1 - \exp\left[-(\lambda_q + \lambda_p)\tau\right]\right\},$$
$$r_k(\xi) = \langle Y_k | 1 \rangle Y_k(\xi), \tag{A.6}$$

and

$$G_0(z,\xi)=\int_{-\infty}^{\infty}dq\,Y_q^{+}(z)Y_q(\xi)= \\ =\delta(z-\xi)-\overline{Y}^{+}(z)\overline{Y}(\xi). \quad (A.7)$$

Eq. (A.7) follows from the completeness of functions X_α and expresses the closure condition of eigenfunctions Y_α. It follows from (A.2), (A.4) and (A.7)

$$A_1(\xi)=-\sigma^2\langle\overline{Y}|1\rangle^2\left\langle\overline{Y}\left|\frac{d\mathcal{E}_0}{dz}\right.\right\rangle^{-1}\frac{d\overline{Y}(\xi)}{d\xi}. \quad (A.8)$$

From Eqs. (1.12) and (1.16) used in conjunction with (A.1), (A.3) and (A.5) one gets in the case of relatively long times $\tau >> \tau_R = \lambda_a^{-1}$,

$$\overline{\Delta\mathcal{E}}^{(2)}(\xi)\approx\int_{-\infty}^{\infty}dkC(k)Y_k(\xi), \quad (A.9)$$

where

$$C(k)=\lambda_k^{-1}\langle Y_k|B(z)\rangle, \\ B(z)=-A_1(z)-\sigma^2w''[\mathcal{E}_0(z)]P_0(z). \quad (A.10)$$

The function $P_0(z)$ in (A.10) is defined by (A.5) if one substitutes $a_{qp}=1$ into (A.5). Thus, the explicit expression of $P_0(z)$ may be derived from (A.5) by taking into account that the integral $\langle Y_k|1\rangle$ in (A.6) is relatively sharp function of k, localized in the interval $|k|<1$. By method of contour integration in the complex y-plane one gets

$$\int_{-\infty}^{\infty}dy\exp(iky)\tanh y=i\pi\sinh(\pi k/2). \quad (A.11)$$

Hence, the integral $\langle Y_k|1\rangle$ in (A.5) is rapidly damped function of k, as seen from the explicit expression of $Y_k(y)$ (2.5). From (A.5) with the help of (A.7) follows

$$P_0(z)\approx(2\lambda_a)^{-1}\left[1-\langle\overline{Y}|1\rangle\overline{Y}(z)\right]^2 \quad (A.12)$$

Substituting (A.8) and (A.12) into (A.10) one gets

$$B(z)=2^{-1}\sigma^2\gamma^2\left\langle\overline{Y}\left|\frac{d\mathcal{E}_0}{dx}\right.\right\rangle\times \\ \times\left\{\langle\overline{Y}|1\rangle^2+\left[1-\langle\overline{Y}|1\rangle\overline{Y}(z)\right]^2\right\}\frac{d\overline{Y}(z)}{dz}. \quad (A.13)$$

In deriving of (A.13) the explicit expressions (2.3) and (2.4) have been used. From (A.9), (A.10) and (A.13) the Eqs. (3.4) and (3.5) follow immediately.

B. It follows from (3.4)

$$\overline{\Delta\mathcal{E}}^{(2)}(\xi)=\int_{-\infty}^{\infty}dk\lambda_k^{-1}b(k)Y_k(\xi), \quad (B.1)$$

where

$$b(k)=\langle Y_k|B\rangle=\int_{-\infty}^{\infty}dz\,Y_k^{+}(z)B(z).$$

The integrand $B(z)$ in (B.1) is the localized function of z with the characteristic length of localization of the order of unity, as follows from Eqs. (3.5) and (2.5). From (B.1) one can see by use of (2.5) that the function $b(k)$ is Fourier transform of the "twin-spike" function, localized in the interval $|z|\le 1$. Note, that the distance between the "spikes" is also of the order of unity. Hence, the dependence $b(k)$ is an oscillating function of k, rapidly (exponentially) damped outside the interval $|k|\le 1$. Evidently, the period of the oscillations is also of the order of unity. Taking into mind that the eigenvalue λ_k in (B.1) is a slowly varying function of k, one can evaluate the field $\overline{\Delta\mathcal{E}}^{(2)}(\xi)$ in (B.1) by expanding of λ_k^{-1} into power series. Retaining the leading term of expansion one substitutes $\lambda_k^{-1}\approx\lambda_a^{-1}$ into (B.1). Thus, by use of the closure condition (A.7) one gets

$$\overline{\Delta\mathcal{E}}^{(2)}(\xi)=\lambda_a^{-1}\int_{-\infty}^{\infty}dzB(z)\int_{-\infty}^{\infty}dk\,Y_k^{+}(z)Y_k(\xi)= \\ =\lambda_a^{-1}\left[B(\xi)-\langle\overline{Y}|B\rangle\overline{Y}(\xi)\right]$$

(B.2)

C. Starting from (3.5) we define the function

$$G_n(z,x)=\int_{-\infty}^{\infty}dk\lambda_k^{-n}Y_k^{+}(z)Y_k(x). \quad (C.1)$$

Let us define the auxiliary function

$$\Gamma_0(z,x;\mu)= \\ =\int_{-\infty}^{\infty}dk(k^2+1)(k^2+\mu)^{-1}Y_k^{+}(z)Y_k(x). \quad (C.2)$$

From (C.1) and (C.2) by use of (2.5) follows

$$G_n(z,x)=(-\lambda_a)^{-n}(n!)^{-1}\frac{\partial^n}{\partial\mu^n}\Gamma_0(z,x;\mu=1). \quad (C.3)$$

Thus, the dependence $G_n(z,x)$ is fully determined by the auxiliary function $\Gamma_0(z,x;\mu)$. It follows from (C.2) and (2.5)

$$\Gamma_0(z,x;\mu)=(2\pi)^{-1}S(x)S^{-1}(z)\times$$
$$\times\int_{-\infty}^{\infty}dkg_0(k;z,x,\mu)\exp[ik(x-z)],$$
$$g_0(q;z,x,\mu)=1+(k^2+\mu)^{-1}(\tanh z\tanh x-\mu)-$$
$$-ik(k^2+1)^{-1}(\tanh z-\tanh x). \quad (C.4)$$

From (C.4) is seen that

$$\Gamma_0(z,x;\mu)=S^{-1}(z)S(x)\{\delta(z-x)+\gamma_0(z,x)$$
$$(\tanh z\tanh x-\mu)-\gamma_1(z,x)(\tanh z-\tanh x)\}, \quad (C.5)$$

where

$$\gamma_0(z,x;\mu)=(2\pi)^{-1}\int_{-\infty}^{\infty}dk(k^2+\mu)^{-1}\exp[ik(x-z)]=$$
$$=2^{-1}\mu^{-1/2}\exp\{-\mu^{-1/2}|z-x|\}, \quad (C.6)$$

$$\gamma_1(z,x;\mu)=i(2\pi)^{-1}\int_{-\infty}^{\infty}dkk(k^2+\mu)^{-1}\exp[ik(x-z)]=$$
$$=\frac{\partial}{\partial x}\gamma_0(z,x;\mu). \quad (C.7)$$

Integration in (C.6) has been evaluated by the method of contour integration in the complex k-plane.
From (C.1) used in conjunction with Eqs. (C.3) and (C.5)-(C.7) follows

$$G_1(z,x)=S(x)[8\lambda_a S(z)\cosh z\cosh x]^{-1}\times$$
$$\times\{\exp(-2z_>)+\exp(-2z_<)-2|z-x|\} \quad (C.8)$$

where $z_>=\max\{z,x\}$ and $z_<=\min\{z,x\}$. Now Eq. (3.8) follows immediately from (C.8) by taking into account that $S(x)\approx S^{-1}\approx 1$, if $\lambda_G>>1$.

D. From definition of translational operator follows

$$\hat{T}(s)=\exp\left(s\frac{\partial}{\partial y}\right)=\sum_{n=0}^{\infty}s^n(n!)^{-1}\frac{\partial^n}{\partial y^n}. \quad (D.1)$$

From (1.16) used with Eqs. (2.2) and (2.6) one gets

$$<<s^{2r+1}>>=0,\ <<s^{2r}>>=$$
$$=(r-1)!!<<s^2>>^r,\ r=1,2,3. \quad (D.2)$$

Now, from (D.1) and (D.2) it is easy to see that

$$<<\hat{T}(s)>>=\hat{d}(s)=\exp\left(2^{-1}<<s^2>>\frac{\partial^2}{\partial y^2}\right). \quad (D.3)$$

From (D.3) the needed Eqs. (4.2) and (4.3) follow at once.

E. It follows from (4.5) that

$$\overline{\Delta\mathcal{E}}(y,\tau)=\int_{-\infty}^{\infty}dk\langle Y_k|1\rangle\hat{R}_kY_k(y), \quad (E.1)$$

where operator $\hat{R}_k$ is defined as follows

$$\hat{R}_k=\int_0^{\tau}dt\exp[\lambda_k(t-\tau)]<<\delta J(t)\hat{T}[s(\tau)]>>. \quad (E.2)$$

Performing the averaging in (E.2) with the help of (2.6) and (D.1), one gets

$$\hat{R}_k=\lambda_k^{-1}K_k(\tau)\frac{\partial}{\partial y}\exp\left[2^{-1}<<s^2(\tau)>>\frac{\partial^2}{\partial y^2}\right], \quad (E.3)$$

where

$$K_k(\tau)=\lambda_k\int_0^{\tau}dt\exp[\lambda_k(t-\tau)]<<\delta J(t)s(t)>>=$$
$$=2\sigma^2\langle\overline{Y}|1\rangle\left\langle\overline{Y}\left|\frac{\partial\mathcal{E}_0}{\partial y}\right.\right\rangle^{-1}[1-\exp(-\lambda_k\tau)]. \quad (E.4)$$

From (E.1), by use of (E.3), (E.4) and (4.3), the Eq. (4.6) immediately follows.

F. Expressions (5.6) - (5.10) are deduced from (4.9) by taking into account the following relation:

$$\mathcal{E}_0(q)=\overline{F}\delta(q)\pm i\gamma^{-1}\left[(q-i\,\mathrm{signy})\cosh\left(\frac{\pi q}{2}\right)\right]^{-1}, \quad (F.1)$$

which may be obtained by the method of contour integration in complex plane. One gets from (4.9) and (F.1)

$$\overline{\mathscr{E}}_0(y,b)=\overline{F}\mp i\gamma^{-1}J(y,b),$$

$$J(y,b)=\int_{-\infty}^{\infty}dq\left[\left(q-i\,\mathrm{sign}\,y\right)\cosh\left(\frac{\pi q}{2}\right)\right]^{-1}\times \quad (F.2)$$

$$\times\exp\left(-bq^2+iqy\right).$$

The integrand $\exp(-bq^2)$ in (F.2) is rapidly damped function q, if $b>>1$. Expanding the slowly varying function,

$$\cosh^{-1}\left(\frac{\pi q}{2}\right)=1+\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n)!}\left(\frac{\pi}{2}\right)^{2n}\mathscr{E}_n q^{2r}, \quad (F.3)$$

and substituting (F.3) into (F.2) one gets

$$J=J_0+J_\Sigma \quad (F.4)$$

where

$$J_0=\int_{-\infty}^{\infty}dq\left[\left(q-i\,\mathrm{sign}\,y\right)\right]^{-1}\exp\left(-bq^2+iqy\right)=$$

$$=i\pi\Phi\left(\frac{y}{2\sqrt{b}}\right) \quad (F.5)$$

$$J_\Sigma=\sum_{m=0}^{\infty}\frac{(-1)^{m+1}}{[2(m+1)]!}\left(\frac{\pi}{2}\right)^{2(m+1)}\mathscr{E}_{m+1}J_m(y,b).$$

Here $\mathscr{E}_n$ is Euler number, $\Phi(x)$ denotes Frensel integral and $J_m(y,b)$ is the following integral:

$$J_m=\int_{-\infty}^{\infty}dq\,q^{2m+1}\exp\left(-bq^2+iqy\right)=$$

$$=\left(\frac{i}{2}\right)\pi^{1/2}(-1)^m y\frac{\partial^m}{\partial b^m}\left[b^{-3/2}\exp\left(-\frac{y^2}{4b}\right)\right]. \quad (F.6)$$

From (F.2) and (F.4) - (F.6) the expressions (5.6) - (5.10) follow immediately.

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Воздействие случаиной силы на домен Ганна

Ф.Г. Басс, Р. Баканас

Рассматривается распространение ганновских слоев (ГС) - нелинейных волн электронной плотности в полупроводниковом образце - под влиянием флуктуаций тока, характеризуемых белым гауссовым шумом. Развита итерационная схема для ряда теории возмущений с целью изучить статистические свойства ГС в гидродинамическом приближении. Исследованы усредненные характеристики ГС для двух статистических ансамблей, описывающих существенно различные физические ситуации. Показано, что флуктуации тока существенно влияют на форму среднего поля ГС, оставляя неизменными в обоих случаях среднюю скорость ГС и падение напряжения на образце. Выведено диффузионное уравнение, описывающее размытие среднего профиля ГС в ансамбле ГС со случайно распределенными фазами. Приведены численные оценки.

Дія випадкової сили на домен Ганна

Ф. Басс, Р. Баканас

Розглянуто поширення ганновських шарів (ГШ) - нелінійних хвиль електронної густини у напівпровідникових зразках - під впливом флуктуацій струму, які характеризуються білим гаусівським шумом. Розвинено ітераційну схему для ряду теорії збурень для вивчення статистичних властивостей ГШ у гідродинамічному наближенні. Досліджено усереднені характеристики ГШ для двох статистичних ансамблів, що описують істотно різні фізичні ситуації. Показано, що флуктуації струму істотно впливають на форму середнього поля ГШ, залишаючи незмінними в обох випадках середню швидкість ГШ і падіння напруги в зразку. Виведено дифузійне рівняння , яке описує розмиття середнього профілю ГШ у ансамблі ГШ з випадково розподіленими фазами. Приведено числові оцінки.