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РАДИОФИЗИЧЕСКИЕ АСПЕКТЫ РАДИОЛОКАЦИИ, РАДИОНАВИГАЦИИ, СВЯЗИ И ДИСТАНЦИОННОГО ЗОНДИРОВАНИЯ

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DEALIASING DOPPLER SPECTRA IN METEOROLOGICAL RADARS

A method of dealiasing of Doppler spectra is proposed. The method is based on the fact that overlaid spectral components of the received signals are statistically independent, and that there is no overlay in range. For implementation of the method, the radar is supposed to transmit trains of pulses with two or more pulse repetition frequencies. In contrast to usual dual-PRF techniques, it does not require the Doppler spectrum to be narrow. The method is validated on experimental data from a meteorological radar, the dealiased spectra are compared with those measured directly at higher pulse repetition frequency.

Key words: meteorological radar, Doppler spectra, aliasing

1. Introduction

Any designer of a pulsed radar system faces the problem of choosing a pulse repetition frequency (PRF) value. Higher PRF values provide a wider range of unambiguous Doppler velocity measurement, while lower values give a wider range of unambiguous measurement of slant distance. Generally, when no additional measures are taken, the maximal unambiguous slant distance $R_{\max}$ and maximum unambiguous velocity $v_{\max}$ are limited by the following relation

$$R_{\max} v_{\max} = c\lambda/8, \qquad (1)$$

where c is the light speed, and λ is the wavelength used.

To overcome this limit, different methods have been proposed. The most widely used ones are based on the staggered pulse repetition time [1, 2], or on the dual pulse repetition frequency [3–5]. These techniques allow estimating the average radial speed for essentially larger $v_{\max}$ values than is predicted by (1). The accuracy of such estimation, however, depends on the spectrum width, effectively implying that the spectrum width should be essentially less than the PRF value.

The large number of existing methods reflects the complexity of the problem and, probably, impossibility of a universal solution applicable to any radar design. This paper gives yet another method of dealiasing Doppler spectra, which is based on some other assumptions than those mentioned above. The most important assumption in the proposed method is that different components of Doppler spectrum are statistically independent. No assumptions regarding the shape of the power spectrum or its continuity are used. Similarly to dual-PRF techniques, this method is based on transmitting trains of pulses at different pulse repetition frequencies with all of them providing the needed value of $R_{\max}$.

The paper has the following structure. Description of the proposed method is given in Section 2. In Section 3, validation of the method is given. Some problems peculiar to the method and options for their solutions are discussed in Section 4. Section 5 discusses possibility of using the method with more than two PRFs values, and gives example of dealiasing of three spectra measured at three different PRFs. Finally, conclusions are given in Section 6.

2. Description of the Method

Measuring Doppler spectra by analyzing reflections in pulsed radars is like analyzing a sampled complex signal with the sampling frequency being equal to the pulse repetition frequency. Let us consider what happens with a continuous complex signal during sampling when the sampling frequency f_S is less than the signal bandwidth. If a signal component has frequency f outside the unambiguous bandwidth $(-f_S/2,\ f_S/2)$, after sampling its frequency appears to be within the mentioned range, modified as

$$f' = f + mf_S, \tag{2}$$

where m is integer. If we calculate the spectrum of the sampled signal, the complex amplitude at the frequency f' is equal to sum of the complex amplitudes at all frequencies f satisfying (2).

In the case of weather radars, the reflected signal from a single range bin can be assumed random. And, actually, the point of interest of meteorologists is not the signal itself but the average Doppler spectrum at a given range bin. In this paper, it is assumed that signal components with frequencies f_1 and f_2 separated by an integer number of PRF ($f_1 = f_2 + mf_{PRF}$) are statistically independent. In this case, the average value of power in the aliased spectrum at a frequency of f is a sum of average powers of the original spectrum at several frequencies:

$$s_{ALIASED}(f) = \sum_m s_{ORIGINAL}(f + mf_{PRF}). \tag{3}$$

This relation can be conveniently described in a matrix form. In practice, the Doppler spectrum is usually calculated as a discrete Fourier transform of the reflected signal. Let us calculate the power of each Fourier component and use the obtained values to compose a vector $\vec{\mathbf{s}}_1$ which is referred below as a measurement vector. Then, let us sample power of the original spectrum of the signal with a frequency interval of $\Delta f = f_{PRF1}/N_1$, which is the same as in the Doppler spectrogram. Here, N_1 is the number of samples in DFT. These samples are also combined in a vector $\vec{\mathbf{s}}$. Now, the effect of sampling on the spectrum can be described as

$$\vec{\mathbf{s}}_1 = \mathbf{M}_1 \cdot \vec{\mathbf{s}}. \tag{4}$$

Here, $\mathbf{M}_1$ is a matrix which in what follows is referred to as a measurement matrix. The number of rows in this matrix is equal to the number of samples used in the discrete Fourier transform (N_1). The number of columns reflects the width of the full spectrum. The matrix is of block-diagonal type. Most of its components are zeros, and only some are units. Here is an example of a measurement matrix, which describes sampling of a signal when the signal bandwidth can be twice as large as the sampling frequency, and 4-samples DFT is used:

$$\mathbf{M}_1 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

As is seen, in this case two elements in each row are units. This reflects the fact that due to aliasing, each component of the spectrogram is a sum of two spectrum components, as was expressed in (3). In terms of relation (3), the units in the middle part of the matrix correspond to $m = 0$, while the units in the top right and bottom left corners correspond to $m = -1$ and $m = 1$, respectively.

In practice, the vector $\vec{\mathbf{s}}_1$ in equation (4) is what we have measured and the vector $\vec{\mathbf{s}}$ is what we need to obtain. It would be convenient to solve the equation by finding the inverse of the matrix $\mathbf{M}_1$:

$$\vec{\mathbf{s}} = \mathbf{M}_1^{-1} \cdot \vec{\mathbf{s}}_1.$$

However, the rank of the matrix $\mathbf{M}_1$ is smaller than the number of its columns and, therefore, a unique solution cannot be found. One of the ways to have a unique solution is to decrease the number of columns to the rank of the measurement matrix, but this would mean that the bandwidth of the original spectrum be less than the sampling frequency. In order to provide a single solution when the original spectrum is wider than the measured one, the rank of the matrix can be increased by adding more rows. In this paper, this is made by measuring one more vector $\vec{\mathbf{s}}_2$ using another sampling frequency f_{PRF2}. For convenience, the second vector should be calculated with another number of samples N_2 in order to have the same frequency interval Δf as in the vectors $\vec{\mathbf{s}}_1$ and $\vec{\mathbf{s}}$. This can be satisfied provided that

$$f_{PRF2}/N_2 = f_{PRF1}/N_1. \tag{5}$$

The two measurement vectors can be combined in one vector $\vec{\mathbf{s}}_M = \begin{pmatrix} \vec{\mathbf{s}}_1 \\ \vec{\mathbf{s}}_2 \end{pmatrix}$, and the measurement matrix for this vector will be $\mathbf{M} = \begin{pmatrix} \mathbf{M}_1 \\ \mathbf{M}_2 \end{pmatrix}$. If N_1 and N_2 do not have common multipliers, the rank of the combined measurement matrix will be equal to $N_1 + N_2 - 1$. This allows the number of columns to be greater than N_1 and N_2, which means that the bandwidth of the original spectrum can be greater than any of the two pulse repetition frequencies. It will be observed that the measurement matrix is not square and, therefore, a pseudoinverse of the measurement matrix should be used. Provided that the length of the original spectrum is equal to the rank of the measurement matrix, the original spectrum can be calculated as

$$\vec{\mathbf{s}} = (\mathbf{M}^* \cdot \mathbf{M})^{-1} \cdot \mathbf{M}^* \cdot \vec{\mathbf{s}}_{\mathbf{M}},$$

where asterisk denotes conjugate transpose of the matrix.

For implementation in radars, it is convenient to have the matrices multiplied before the radar starts its operation:

$$\mathbf{A} = (\mathbf{M}^* \cdot \mathbf{M})^{-1} \cdot \mathbf{M}^*.$$

Then, during the radar operation, dealiasing of the measured spectra can be done through a single matrix-vector multiplication:

$$\vec{\mathbf{s}} = \mathbf{A} \cdot \vec{\mathbf{s}}_{\mathbf{M}}. \qquad (6)$$

Summing up, the practical implementation should be as follows. The transmitted signal should consist of trains of pulses with two different PRFs. That is, N_1 pulses with pulse repetition frequency of f_{PRF1} should be followed by N_2 pulses with PRF of f_{PRF2}. For each range bin, radar returns measured during the first train of pulses are used to calculate the vector $\vec{\mathbf{s}}_1$ and the ones measured during the second train are used to calculate the vector $\vec{\mathbf{s}}_2$. The complete measurement vector $\vec{\mathbf{s}}_M = \begin{pmatrix} \vec{\mathbf{s}}_1 \\ \vec{\mathbf{s}}_2 \end{pmatrix}$ is then used to calculate the dealiased spectrum in accordance with (6).

3. Validation of the Method

In order to verify the validity of the assumptions made when deriving the method of spectrum dealiasing, raw radar data were used. The data were recorded with a 36 GHz Doppler meteorological radar during its normal operation. Namely, it was the MIRA-36 radar manufactured by the Institute of Radio Astronomy, Kharkiv, Ukraine and METEK GmbH, Elmshorn, Germany [6, 7]. During the radar operation, the pulse repetition frequency was 9 kHz. Signal for the method validation was taken from a single range bin. Its central Doppler frequency was about 2.5 kHz and its bandwidth about 900 Hz. Before using the data for validation, they have been multiplied by a complex harmonic signal with frequency of –2.5 kHz in order to shift the whole spectrum to low frequencies. Then, measurement with lower values of the pulse repetition frequency was modeled by decimating the original signal by some decimation factor N_{DEC} (an integer value). This was equivalent to having the signal recorded with a PRF value of $9/N_{DEC}$ kHz. The decimated signals were then analyzed as described in Section 2. Thus, three spectra were obtained: two aliased spectra corresponding to the two PRFs, and a dealiased spectrum calculated from them.

In order to obtain a benchmark spectrum for comparison, the original signal was decimated choosing such value of N_{DEC}, that the total bandwidth were the same as in the dealiased spectrum.

In Fig. 1, one can see two aliased spectra obtained for PRF values of 500.0 and 529.4 Hz (decimation factors used were 18 and 17, respectively). The numbers of pulses per train were 170 and 180, respectively. In Fig. 2, a comparison between the dealiased and benchmark spectra is given. The benchmark spectrum was calculated using decimation factor of 9 with 340 samples DFT.

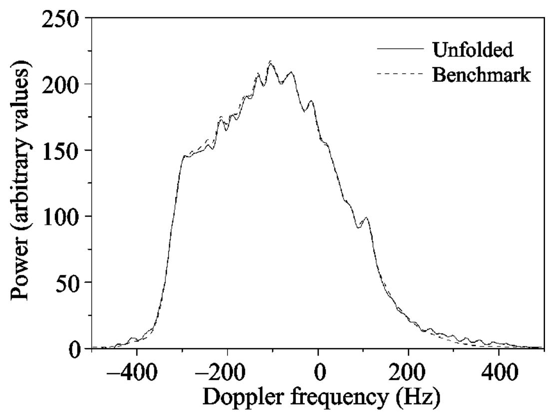

Fig. 1. Aliased spectra corresponding to PRF values of 500.0 and 529.4 Hz (decimation factors 18 and 17, respectively). The numbers of pulses per train are 170 and 180, respectively. Both spectra are results of averaging of 171 spectrograms

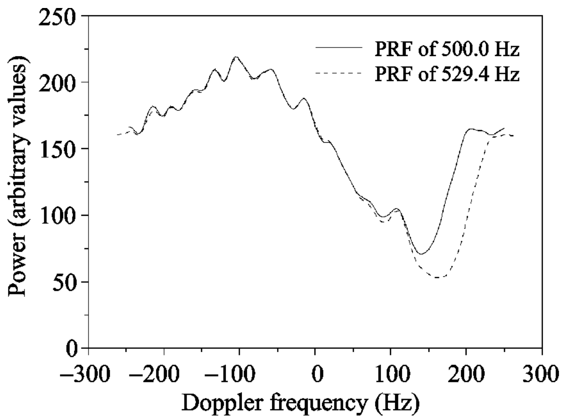

Fig. 2. Comparison of the dealiased spectrum with the benchmark. The benchmark spectrum is calculated using decimation factor of 9 and the DFT length is 340 samples

As is seen from Fig. 2, the two spectra are very close that proves applicability of the proposed method to meteorological Doppler measurements. Some difference observed between the two spectra has its origin in random fluctuations of the spectrum power around its average value, which will be discussed below.

4. Implementation Problems and Possible Solutions

The proposed method has some problems which need to be addressed when implementing the method in practical systems. First, there is a problem of Discrete Fourier Transform. The most common implementations of DFT are Fast Fourier Transform algorithms which require the number of samples to be a power of 2. These algorithms cannot be used with the proposed method because if both of the measurement sub-vectors $\vec{\mathbf{s}}_1$ and $\vec{\mathbf{s}}_2$ have lengths equal to some powers of 2, the rank of the measurement matrix would be equal to the length of the larger sub-vector, that is there would be no gain in spectrum bandwidth. At least three solutions to this problem are possible.

1) Fast Fourier Transform algorithms for numbers of samples other than powers of 2 are possible, though less common.

2) With the rapid increase of digital processing boards capabilities, it is possible that slower DFT algorithms would be acceptable.

3) Though in Section 2 it was insisted that the frequency resolution of all spectra should be the same (resulting in requirement (5) which implies restriction on the choice of N_1 and N_2), actually it is possible to generalize the method for the case of different frequency resolutions of the measured spectra. In this paper, the restriction on frequency resolution (5) is made rather for the sake of convenience of explanation of the method, and in order to avoid interpolations of spectra when deriving a correct measurement matrix.

The second problem of method implementation is increase of noise in the dealiased spectrum as compared to the original spectra. The source of this problem is the fact that the method was derived for average spectral powers. In practice, we always have some fluctuations around the average values, and these fluctuations increase during dealiasing. So far, the only solution to this problem is averaging of the measured spectra. Averaging can be done either in time (by accumulating the spectra during several repeats of pulse trains), or in frequency (smoothing the spectra with some kind of filtering technique), or both. The results in Section 3 are obtained using averaging both in time and in frequency. In time, 171 pulse trains of each PRF were analyzed, so both vectors were obtained by averaging 171 spectra.

5. Using More than Two Pulse Repetition Frequencies

In Section 2, it has been described how to dealias spectra measured at two different PRFs. When two PRFs are used, the bandwidth of the dealiased spectrum is usually two times larger than the lesser of the two PRFs. Compared to the larger PRF, the bandwidth is only 1.6–1.9 times larger. It is possible to enhance this value by using more than two PRFs. For this, measurement vector and measurement matrix are composed as

$$\mathbf{s}_{\mathbf{M}} = \begin{pmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \dots \\ \mathbf{s}_{\mathbf{N}} \end{pmatrix}, \qquad \mathbf{M} = \begin{pmatrix} \mathbf{M}_1 \\ \mathbf{M}_2 \\ \dots \\ \mathbf{M}_{\mathbf{N}} \end{pmatrix}.$$

It will be observed, however, that using many PRFs with close values makes the dealiased spectrum even more distorted with fluctuations than in the case of two PRFs. Averaging spectra in time also becomes less efficient: the maximum time during which spectra can be averaged is limited by the dynamics of atmosphere, and the larger is the number of PRFs we use, the more time it takes to transmit and receive all trains of pulses. Also, the values of PRF should be chosen carefully so that all columns of the final measurement matrix would not be linearly dependent.

Figs. 3 and 4 show an example of using pulse trains with three different PRFs. This example was calculated using the same experimental data as in Section 3. Fig. 3 shows the measured spectra. The PRF values used for this calculation are 333.3, 375.0, and 428.6 Hz. The corresponding decimation factors are 27, 24, and 21. The numbers of pulses in the trains are 168, 189, and 216. To reduce the effect of spectrum fluctuations, 77 pulse trains of each PRF were analyzed.

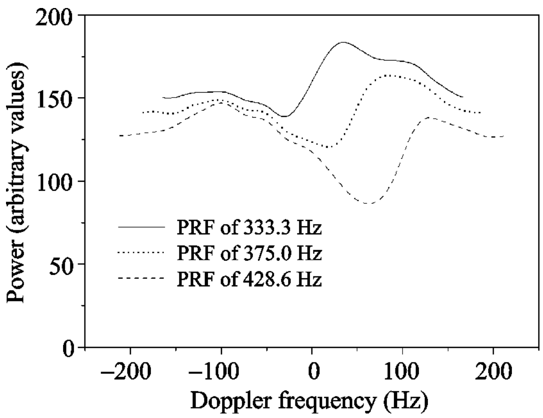

Fig. 3. Aliased spectra corresponding to PRF values of 333.3, 375.0, and 428.6 Hz (decimation factors 27, 24, and 21, respectively). The numbers of pulses per train are 168, 189, and 216, respectively. Each of the spectra is a result of averaging of 77 spectrograms

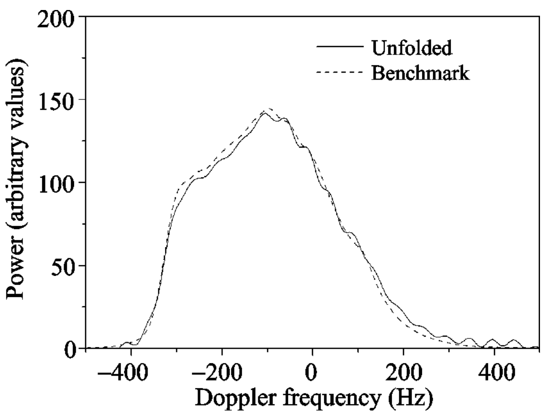

Fig. 4. Comparison of the dealiased spectrum with the benchmark spectrum. The benchmark spectrum is calculated using decimation factor of 9, the DFT length being 504 samples

Fig. 4 shows comparison of the dealiased spectrum with the benchmark one. The benchmark spectrum was calculated using the decimation factor of 9 with 504 samples DFT. The shape of the spectrum is reconstructed correctly, even though the measured spectra had more severe aliasing than in the previous example. One may notice that spectra in Fig. 4 are less detailed than those in Fig. 2. This is a result of stronger averaging in frequency, which was needed to attenuate the effect of fluctuations.

6. Conclusions

A method of dealiasing Doppler spectra measured by meteorological radars is proposed. It is based on transmitting trains of pulses with different values of pulse repetition frequency. The difference between the spectra of received signals allows restoring (dealiasing) of the original spectrum. The method uses no assumptions regarding the shape of the power spectrum except that it fits within the dealiased spectrum bandwidth. The theoretical upper limit of relation between the spectral width and PRF values is equal to the number of PRF values used.

The proposed method has been validated using raw data from a meteorological radar. Some problems of method implementation have been discussed and solutions to these problems proposed.

The proposed method can be used for dealiasing of spectra not only in meteorological radars, but also in any system where the analyzed signal is random and only the average spectrum is wanted.

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ВОССТАНОВЛЕНИЕ ДОППЛЕРОВСКИХ СПЕКТРОВ В МЕТЕОРОЛОГИЧЕСКИХ РАДИОЛОКАТОРАХ

Предложен метод восстановления допплеровских спектров. Метод основан на том, что наложенные спектральные компоненты принятых сигналов статистически независимы, а также на отсутствии наложений сигнала по дальности. Для реализации метода локатор должен излучать пачки импульсов на двух или более частотах повторения импульсов. В отличие от обычных методов, основанных на двух частотах повторения импульсов, он не требует узости допплеровского спектра. Метод проверен на экспериментальных данных, записанных метеорологическим локатором; восстановленные спектры сравниваются со спектрами, измеренными на большей частоте повторения импульсов.

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ВІДНОВЛЕННЯ ДОППЛЕРІВСЬКИХ СПЕКТРІВ У МЕТЕОРОЛОГІЧНИХ РАДІОЛОКАТОРАХ

Запропоновано метод відновлення допплерівських спектрів. Метод базується на тому, що накладені спектральні компоненти прийнятого сигналу є статистично незалежними, а також на відсутності накладання сигналу за відстанню. Для реалізації методу локатор має випромінювати пакети імпульсів на двох або більше частотах повтору імпульсів. На відміну від звичайних методів, що використовують дві частоти повтору імпульсів, він не вимагає малої ширини допплерівського спектру. Метод перевірено на експериментальних даних, записаних метеорологічним локатором; відновлені спектри порівнюються зі спектрами, отриманими на більшій частоті повтору імпульсів.

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