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Multielement Linear Interferometers with One Remote Element

L. E. Kopilovich

Institute of Radio Physics and Electronics, NAS Ukraine,
12, Akad. Proskura St., Kharkiv, 61085, Ukraine
E-mail: kopil@ire.kharkov.ua

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New schemes of constructing multielement linear interferometers based on the difference sets are suggested. It is shown that adding a remote element to the earlier model [3] allows to extend a completely covered spatial-frequency range.

Introduction

Supersynthesis is the method for observing cosmic sources which is realized using multielement linear radio interferometers [1]. The best known systems of such a type are the Cambridge series telescopes [2].

Optimization of the linear interferometer structure aims at providing a complete coverage of the maximum spatial-frequency range (i. e. recording all spectral components of the signal in this range) for the fixed number of elements. In practice, when the number of elements is large, their arrangement is found empirically, thus, being not optimum; besides, the resultant coverage has gaps.

To build the best covering system, it was suggested in [3] to employ the construction based on difference sets [4, 5] possessing essentially better characteristics than the operating telescopes with a linear arrangement of elements. Note that such a system has a certain redundancy that decreases the number of the obtained baselines as compared with the nonredundant equielement one.

Here, the systems built using the difference sets which provide an extended range of complete coverage due to adding a properly suited remote element is studied.

Nonredundant systems with a remote element

First, consider nonredundant systems based on the difference sets. Recall that a cyclic difference set (CDS) with the parameters V, k, λ is a k-element integer set $\{d_j\}$, $0=d_l<d_2<...<d_k<V$, such that any integer μ, $0<\mu<V$ has exactly λ representations of the form

$$\mu \equiv d_i - d_j (\mathrm{mod}\, V), \qquad (1)$$

i. e. in the form $d_i - d_j$ or $V-(d_i - d_j)$ [6]. Hence, for $\lambda=1$ such a set is nonredundant. The CDSs with $\lambda=1$ exist at all $k=q+1$, where q is a power of a prime, and in every such a case there exists an ensemble of such sets [6]. Note that for these CDSs $V=k^2-k+1$. Hereafter, when talking on CDSs we mean the sets with $\lambda=1$. Such sets provide coverage of the section of the length $V-d_k-1$, and after the missed point $V-d_k$, there is again the covered section up to $V-d_{k-1}-1$ or $V-(d_k-d_2)-1$.

The set with $d_{k\,\min}=\min(d_k)$ taken from the ensemble of k-element CDSs provides covering the section of maximum length $n_1 = V - d_{k\,\min} - 1$. The quantities n_1 can be found from the table of the maximally compressed CDSs given in [7]. Note that these CDSs are not necessarily the shortest ones among various nonredundant sets; however, as can be seen from the available data at $k \le 20$ [8], the sets which are not CDSs provide a complete coverage of substantially smaller sections.

The results yielded can be essentially improved by adding the element V to a k-element CDS. At this point, the widened set remains nonredundant, since to the system of differences $\{d_i - d_j\}$ only new differences of the form $V-d_j = V-(d_j-d_1)$ are added. In this case, the points $V-d_k$ and $V-d_{k-1}$ are within the covered section, thus the whole section up to $V-(d_k-d_2)-1$ becomes covered. To maximize the length of this section, now one must minimize the quantity d_k-d_2; to this end, its maximum length is $n_2=V-(d_k-d_2)_{\min}-1$.

The set having the minimum value of d_k-d_2 should be sought among the k-element CDSs, all of which been obtainable from the initial one by multiplying its elements by various integers coprime with V and then by cyclically shifting it modulo V. As for the initial set, it can be taken from the CDS table in [6] or [7]. Note that sometimes d_k in such a set is equal to $d_{k\min}$, and always $(d_k-d_2)_{\min}<d_{k\min}$.

Both cases considered are illustrated in Fig. 1 for $k=8$. In the first case, the CDS $\{d_j\}$ is maximally compressed, whereas in the second one only the part between d_2 and d_k is compressed. Here both sets are equal in length and $n_1=21$, $n_2=28$.

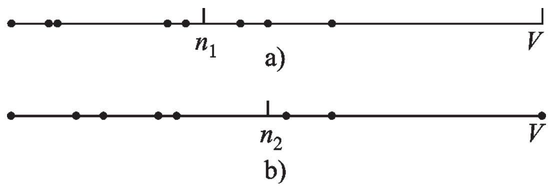

Fig. 1. *Arrangement of interferometer elements. The added element is placed at the point V. The maximum covered spatial-frequency range with no element at V is [0,n_1], and [0,n_2] in the opposite case. Here,* $n_1=V-d_{k\,min}-1$, $n_2=V-(d_k-d_2)_{min}-1$

The joint set consisting of a k-element CDS and the added point V often covers a larger section than even a CDS with more elements does. So, e. g. the 18-element CDS

{0, 35, 57, 63, 76, 97, 115, 124, 135, 140, 166, 189, 190, 219, 222, 226, 234, 236}

plus the added point $V=307$ provides covering the section of the length $n_2=105$, whereas the compressed 20-element CDS

{0, 1, 8, 11, 68, 77, 94, 116, 121, 156, 158, 179, 194, 208, 212, 228, 240, 253, 259, 283}

covers the section of the length $n_1=97$; for the CDS with $k=30$ plus $V=871-$ $n_2=247$, whereas at $k=33-$ $n_1=197$, and so on.

The data on the best coverage provided by the sets of both types, with $k\le 100$, are given in Table 1. One can see that in most cases the value of n_2 ensured by a k-element CDS exceeds that of n_1 for the CDS with the nearest greater element number.

Table 1. *The characteristics of the interferometers employing cyclic difference sets; k and V are the set parameters (k being the number of the set elements;* $V=k^2-k+1$); n_1 *and* n_2 *denote the maximum length of a completely covered spatial-frequency range when the additional element in the point V is either absent or present, respectively*

k	V	n_1	n_2
9	73	27	37
10	91	35	45
12	133	47	60
14	183	55	68
17	273	71	91
18	307	90	105
20	381	97	134
24	553	127	180
26	651	158	170
28	757	173	203
30	871	190	247
32	993	208	245
33	1057	197	245
38	1407	260	307
42	1723	325	417
44	1893	296	379
48	2257	341	426
50	2451	356	492
54	2863	489	515
60	3541	521	629
62	3783	567	598
65	4161	567	633
68	4557	600	737
72	5113	634	782
74	5403	649	889
80	6321	757	912
82	6643	711	886
84	6973	813	952
90	8011	1033	1232
98	9507	997	1149

The systems described provide no-gap coverage of the range close to that ensured by the operating telescopes having nearly the same element number; besides, owing to nonredundancy, they form a series of additional baselines.

Systems formed by several CDSs with one element added

Consider the construction described in [3] which consists of four CDSs of similar structure arranged on segments $[mV, mV + d_k]$, $m = 0, 1, 4, 6$. Here the differences between the elements of the same set are repeated four times, while those between the elements of different sets are unique. Such a system provides a complete coverage of the section of the length $7V - d_k - 1$, and the length $L_1 = 7V - d_{k\,\mathrm{min}} - 1$ is close to the maximum attainable one. Nevertheless, it can be increased to $L_2 = 7V - (d_k - d_2)_{\mathrm{min}}$ by adding an element d which can be chosen in several ways. If $d = 7V$ is taken, then few new differences would appear; if $d = 8V$ or $11V$, the number of those would increase, and with $d = 13V$ all appearing differences in a widened system would be new (the interferometer scheme considered in the latter case is shown in Fig. 2).

Fig. 2. *The scheme of a four-section interferometer with one remote element. The element location areas are shaded. The added element is placed at the point d = 13V*

The gain in the length of the maximum covered section, when the element d is added to the built $4k$-element system, is equal to that obtained in Sect. 2 for the k-element system (i. e. CDS) plus the added point V. Thus, the 9-element set with the minimum value of $d_k - d_2$ has the form

$$\{0, 16, 17, 28, 36, 42, 46, 49, 51\},$$

and the system of four similar in structure shifted CDSs plus an element added (in any of the above pointed variants) provides covering the section of the length $L_2 = 475$, whereas the system of four compressed CDSs, the first of which being

$$\{0, 2, 10, 24, 25, 29, 36, 42, 45\}$$

covers the section of length $L_1 = 465$. At $k = 12$ such constructions give the coverage of the sections of the lengths $L_2 = 858$ and $L_1 = 845$, and so on. Thus, here we see that adding one more element is far less efficient than in the previous case.

Conclusion

The results obtained imply that:

– among the considered nonredundant systems $\{d_j\}$ $(j = 1, ..., k)$, the complete coverage of maximum spatial-frequency range is provided by those whose element arrangement obeys the law of a cyclic difference set, with the parameters V, k; $\lambda = 1$, having minimum value of $d_k - d_2$, plus the point V;

– when building a linear interferometer based on the construction consisting of several similar shifted CDSs, the addition of a properly suited remote element also gives a small increase in the length of the covered range.

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Многоэлементные линейные интерферометры с одним отдаленным элементом

Л. Е. Копилович

Предлагаются новые схемы построения многоэлементных линейных интерферометров на основе разностных множеств. Показано, что введение добавочного отдаленного элемента в ранее разработанную модель [3] позволяет увеличить участок полного покрытия диапазона пространственных частот.

Багатоелементні лінійні інтерферометри з одним віддаленим елементом

Л. Ю. Копилович

Пропонуються нові схеми побудови багатоелементних лінійних інтерферометрів на основі різницевих множин. Показано, що введення додаткового віддаленого елемента в попередню модель [3] дозволяє збільшити ділянку повного покриття діапазону просторових частот.