

# Electromagnetic Characteristics of the Simplest Strip System

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The main electromagnetic characteristics of the noncoplanar system of two narrow strips with parallel edges are investigated. They are determined on the basis of the integral equation technique, using the local coordinate systems. Analytical expressions of the longitudinal and transverse surface current densities are presented. They are validated by passing to the limit of a single strip and allow both analytic and numerical examination of the strip interaction. By using the obtained expressions, the field in the far zone, the scattering cross section and the scattering coefficient are deduced. These expressions are sufficiently simple and convenient for numerical calculation.

## 1. Introduction

The systems of strips are widely used in various branches of physics and technology. In particular, they are applied in antenna technology, electronics, technical metrology, nuclear technology, acoustics and so on [1, 2]. Many authors deal with problem of the electromagnetic wave scattering by such structures (see ref. in [1-9]). The aim of this paper is to obtain the main scattering characteristics, such as the field in the far zone, the scattering cross section and the scattering coefficient [1]. The problems of scattering of a plane linearly polarized monochromatic wave by the system of  $N$  noncoplanar strips have been considered in [6, 7]. The asymptotic expressions for the surface current densities have been found for the case when the wave length of an incident wave is large compared to the dimensions of the strip cross-section. The obtained expressions have been validated by passing to the limit of large distance between strips as compared to the strip width and can be directly applied to the calculation of the main electromagnetic characteristics for the narrow strip systems. First of all, we deduce the main electromagnetic characteristics of the simplest two-strip system and examine them.

## 2. The Problem Formulation and General Expressions

Let us consider a system of two noncoplanar flat strips with parallel edges, so that the cross-sectional view (in  $xOz$  plane) presents two segments  $S_m = (-a_m, a_m)$ ,  $m = 1, 2$ , which are located arbitrarily (see Fig. 1). The strips are assumed to be perfectly conducting and absolutely thin. The system is excited by a plane linearly polarized monochromatic wave.

In the case of  $E$ -polarization (the electric vector  $\vec{E}$  is parallel to the strip edges) we have the

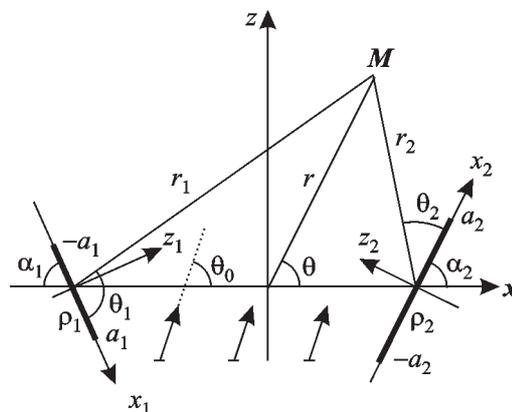


Fig. 1. The cross section of the simplest two-strip system

following expression for the unique component of the electric field strength:

$$E_y = e^{ikr \cos(\theta - \theta_0)} - \frac{i}{4} \sum_{m=1}^2 \int_{-a_m}^{a_m} \phi_m^0(x) H_0^1(kR_m) dx, \quad (1)$$

where  $k$  is the wave number,  $r$  and  $\theta$  are general polar coordinates in the  $xOz$  plane,  $\theta_0$  is the angle between the direction of incident wave and  $x$ -axis,  $H_0^1(z)$  is the Hankel's function (which is the free space Green's function),  $R_m = \sqrt{(x_m - x)^2 + z_m^2}$ ,  $x_mOz_m$  is the local coordinate system with the origin in the center of the  $m$ -th segment. The physical sense of the unknown functions  $\phi_m^0(x)$  is as follows [1]:

$$\phi_m^0(x_m) = \left( \frac{\partial E_{y_m}(x_m, z_m)}{\partial z_m} \right)_{z_m \rightarrow 0+} - \left( \frac{\partial E_{y_m}(x_m, z_m)}{\partial z_m} \right)_{z_m \rightarrow 0-} = -ij_y(x_m),$$

where  $j_y(x)$  is the surface current parallel to the edges of the  $m$ -th strip (the longitudinal current).

In the case of  $H$ -polarization (the vector  $\vec{H}$  is parallel to the strip edges) for the unique component of the magnetic field strength we have the formula:

$$H_y = \frac{i}{k\eta} \left[ e^{ikr \cos(\theta - \theta_0)} - \frac{i}{4} \sum_{m=1}^2 \frac{\partial}{\partial z_m} \int_{-a_m}^{a_m} \psi_m^0(x) H_0^1(kR_m) dx \right], \quad (2)$$

where  $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$ ,  $\mu_0$  and  $\epsilon_0$  are the magnetic and electric constants,  $\psi_m^0(x)$  are the unknown transverse currents in the strips perpendicular to their edges.

The longitudinal and transverse currents are excited by the plane linearly polarized monochromatic electromagnetic wave and themselves excite the scattering electromagnetic fields. To find the unknown functions  $\phi_m^0(x)$  and  $\psi_m^0(x)$  it is

necessary to satisfy the boundary conditions  $E_y|_{S_m} = 0$ . Using these conditions, we deduce two systems of singular integral equations, which are presented in a more general form in [6, 7]. There are many numerical methods for solving such integral equations [8-11], but an asymptotic approach was used in [6, 7] and the following asymptotic expressions were adduced:

$$\phi_m(t) = a_m \phi_m^0(a_m t) = \frac{\phi_{m0} + \chi_m t \pi \phi_{m1}}{\pi \sqrt{1-t^2}} + O(\chi_m^2), \quad (3)$$

$$|t| < 1,$$

$$\psi_m(t) = \psi_m^0(a_m t) = i\chi_m \sqrt{1-t^2} 2M_m \times \sin(\theta_0 - (-1)^m \alpha_m) + O(\chi_m^2), \quad (4)$$

where  $\chi_m = ka_m \ll 1$ ,

$$\phi_{m0} = \frac{-2\pi}{\ln \frac{\gamma \chi_m}{4i}} \frac{M_m - M_{3-m} \epsilon_{3-m}}{1 - \epsilon_1 \epsilon_2}, \quad (5)$$

$\ln \gamma = 0.5772\dots$ ,  $M_m = \exp\left[-i(-1)^m \frac{k\rho}{2}\right]$ ,  $\rho$  is the distance between the centers of the segments; the magnitude  $\epsilon_m$  depends on the main frequency parameters  $\chi_m$  and  $k\rho$  and it is defined by the formula

$$\epsilon_m = -i \frac{\pi}{2} \frac{H_0^{(1)}(k\rho)}{\ln \frac{\gamma \chi_m}{4i}}. \quad (6)$$

To write the expressions (3)-(6) we have chosen a general coordinate system in the following way: the  $x$ -axis passes through the segments' centers, the latter being equidistant from the  $z$ -axis (see Fig. 1).

The second addend in expression (3) is determined by the coefficient  $\phi_{m1}$ :

$$\phi_{m1} = 2iM_m \cos(\theta_0 - \alpha_m) - \frac{i}{2} \phi_{(3-m)0} \cos \alpha_{3-m} H_1^{(1)}(k\rho),$$

here  $\alpha_m$  is the angle between the  $x_m$ -axis and the  $x$ -axis. The expressions (3) and (4) are confirmed by passing to the limit  $\rho \rightarrow \infty$  and are very simple and convenient for numerical calculation. They allow both numerical and analytical examination of the mutual influence between the strips [12, 13].

In particular, using the formula (5), it is easy to obtain the expression

$$\Phi_{10} = \Phi_1 - \Phi_2 \varepsilon_1 + \Phi_1 \varepsilon_1 \varepsilon_2 - \Phi_2 \varepsilon_1^2 \varepsilon_2 + O(\varepsilon^4),$$

where  $\Phi_m = \frac{-2\pi M_m}{\ln(\gamma \chi_m / 4i)}$ ,  $\varepsilon = \max(\varepsilon_1, \varepsilon_2)$ . It describes the process of multiple reflection. Really, the first addend of this asymptotic expression is associated with a separate strip. It is equal to the main term of asymptotic expression for a single strip in case of large wavelengths. The other addends are associated with the mutual influence. The second addend, the main term of interaction, has two factors. The first one is the leading addend for another strip and the second factor is  $\varepsilon_1$  (see expression (6)), which depends on the distance between the strips and tends to zero as the distance increases. Thus, this addend describes the simplest influence of the second strip. The third addend has three factors: the first one is the same as the main term, two another tend to zero at large distances. Hence, it presents the simplest influence the first strip on itself by means of the second strip.

### 3. Scattered Fields in the Far Zone

To examine the electromagnetic field induced by the strips' currents in the far zone, let us suppose that the distance from the system of segments to the point of observation is large as compared to the wavelength of the incident wave. For this reason, we use the following transformations of the coordinates:

$$x_1 = \frac{\rho}{2} \cos \alpha_1 + r \cos(\theta + \alpha_1),$$

$$z_1 = \frac{\rho}{2} \sin \alpha_1 + r \sin(\theta + \alpha_1),$$

$$x_2 = -\frac{\rho}{2} \cos \alpha_2 + r \cos(\theta - \alpha_2),$$

$$z_2 = \frac{\rho}{2} \sin \alpha_2 + r \sin(\theta - \alpha_2)$$

and the asymptotic expressions for the Hankel's function [14]

$$H_0^{(1)}(kR_m) \approx \sqrt{\frac{2}{i\pi kR_m}} \exp[ikR_m],$$

$$\begin{aligned} \frac{\partial}{\partial z_m} H_0^{(1)}(kR_m) &= -H_1^{(1)}(kR_m) \frac{kz_m}{R_m} \approx \\ &\approx \frac{1+i}{\sqrt{\pi kR_m}} \exp[ikR_m] \frac{kz_m}{R_m}. \end{aligned}$$

Returning to the expressions (1) and (2), let us consider only the scattered field. Since

$R_m = r_m - x \cos \theta_m + O\left(\frac{1}{r_m}\right)$ , where  $r_m$  and  $\theta_m$  are the local polar coordinates of the point of observation and  $x$  is variable of integration, we have

$$E_y^{(s)} \approx -\sqrt{\frac{i}{2\pi}} \frac{1}{2} \left[ \frac{e^{ikr_1}}{\sqrt{kr_1}} \hat{\phi}_1(\chi_1 \cos \theta_1) + \frac{e^{ikr_2}}{\sqrt{kr_2}} \hat{\phi}_2(\chi_2 \cos \theta_2) \right],$$

$$\begin{aligned} H_y^{(s)} &\approx \frac{1+i}{4\eta\sqrt{\pi}} \left[ \frac{e^{ikr_1}}{\sqrt{kr_1}} \sin \theta_1 \hat{\psi}(\chi_1 \cos \theta_1) + \right. \\ &\left. + \hat{\phi}_2(\chi_2 \cos \theta_2) \sin \theta_2 \frac{e^{ikr_2}}{\sqrt{kr_2}} \right], \end{aligned}$$

where

$$\hat{\phi}_m = \int_{-1}^1 \phi_m(t) e^{-i\lambda_m t} dt \quad \text{and} \quad \hat{\psi}_m = \int_{-1}^1 \psi_m(t) e^{-i\lambda_m t} dt$$

are the Fourier images of the surface currents in the  $m$ -th strip,  $\lambda_m = \chi_m \cos \theta_m$ .

The vectors of the electromagnetic field then can be represented in the form:

$$E_y^{(s)} \approx \frac{e^{ikr}}{\sqrt{kr}} A(\theta), \quad H_y^{(s)} \approx \frac{e^{ikr}}{\sqrt{kr}} B(\theta),$$

where  $A(\theta)$  and  $B(\theta)$  characterize the dependence on polar angle. Therefore, we should replace every local polar coordinate system  $r_m, \theta_m$  by the general system  $r, \theta$ .

Applying the law of cosines for the triangle  $(r, r_m, \rho_m)$  (see Fig. 1)

$$r_1^2 = r^2 + \rho_1^2 + 2r\rho_1 \cos(\pi - \theta),$$

$$r_2^2 = r^2 + \rho_2^2 + 2r\rho_2 \cos\theta,$$

it easy to see, that

$$r_m = r + (-1)^m \rho_m \cos\theta + O\left(\frac{1}{r}\right) \quad \text{and}$$

$$e^{ikr_m} \approx e^{ikr} e^{ik(-1)^m \rho_m \cos\theta}, \quad \rho_1 = \rho_2 = \frac{\rho}{2}.$$

Thus, we obtain the expressions:

$$A(\theta) = -\frac{1}{2} \sqrt{\frac{i}{2\pi}} \left[ \exp\left(-i \frac{k\rho}{2} \cos\theta\right) \hat{\phi}_1(\chi_1 \cos\theta_1) + \exp\left(i \frac{k\rho}{2} \cos\theta\right) \hat{\phi}_2(\chi_2 \cos\theta_2) \right],$$

$$B(\theta) = \frac{1+i}{4\eta\sqrt{\pi}} \left[ \exp\left(-i \frac{k\rho}{2} \cos\theta\right) \sin\theta_1 \hat{\psi}_1(\chi_1 \cos\theta_1) + \exp\left(i \frac{k\rho}{2} \cos\theta\right) \sin\theta_2 \hat{\psi}_2(\chi_2 \cos\theta_2) \right].$$

It should be noted that for  $kr \gg 1$   $\theta_1 \approx \theta + \alpha_1$  and  $\theta_2 \approx \theta - \alpha_2$ .

Now we investigate the Fourier transforms of the surface currents. In order to find their asymptotic expressions we use the formulas (3) and (4). Performing the transformation, we need to examine the integrals

$$I_k(\chi) = \int_{-1}^1 \exp[i\chi t] \frac{t^k dt}{\sqrt{1-t^2}}. \quad \text{Taking into account}$$

that  $\chi \ll 1$ , we have

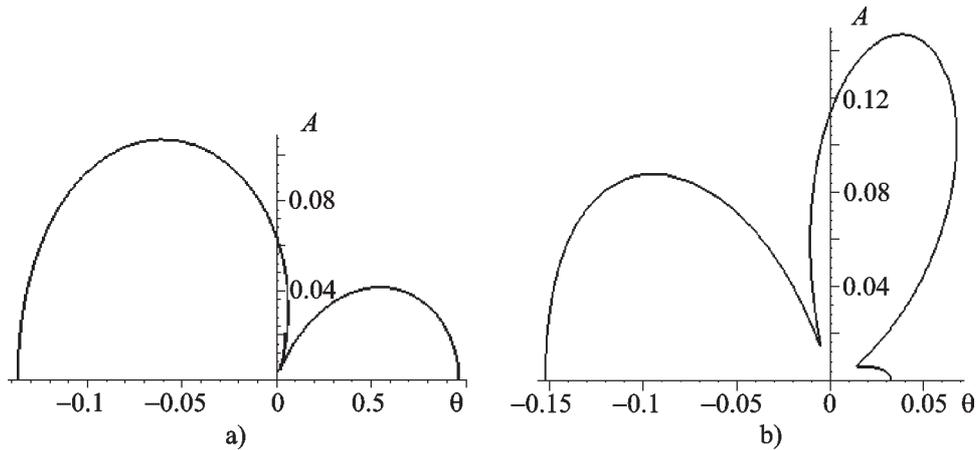
$$I_0(\chi) = \pi + O(\chi^2), \quad I_1(\chi) = O(\chi),$$

$$I_2(\chi) = \frac{\pi}{2} + O(\chi^2), \quad \text{so that}$$

$$A(\theta) = -\frac{1}{2} \sqrt{\frac{i}{2\pi}} \left[ \exp\left(-i \frac{k\rho}{2} \cos\theta\right) \phi_{10} + \exp\left(i \frac{k\rho}{2} \cos\theta\right) \phi_{20} \right] + O(\chi^2), \quad (7)$$

$$B(\theta) = \frac{1+i}{2\eta\sqrt{\pi}} \left[ \chi_1 M_1 \exp\left(-i \frac{k\rho}{2} \cos\theta\right) \times \sin(\theta_0 + \alpha_1) \sin(\theta + \alpha_1) + \chi_2 M_2 \exp\left(i \frac{k\rho}{2} \cos\theta\right) \times \sin(\theta_0 - \alpha_2) \sin(\theta - \alpha_2) \right] + O(\chi^2).$$

These expressions can be successfully used for numerical calculation. As an example, we apply the expression (7) for the system with the frequency parameters  $\chi_1 = 0.1$ ,  $\chi_2 = 0.08$ ,  $k\rho = 2.5$  and  $5$ . The numerical results are presented in Fig. 2, where the dependence of  $|A(\theta)|$  on polar angle  $\theta$  is shown in the polar coordinate system. The values of  $|A(\theta)|$  are displayed using the Cartesian coordinate axes.



**Fig. 2.** Dependence of  $|A(\theta)|$  on polar angle for the system with the frequency parameters  $\chi_1 = 0.1$ ,  $\chi_2 = 0.08$ : a)  $kp = 2.5$ ; b)  $kp = 5$

#### 4. Scattering Cross Section

The scattering cross section is an important characteristic of the scatterer and is defined by the ratio of the scattered energy flux to the energy flux of the exciter's field. The scattered energy flux is given by the integral of the energy flux density vector (Umov-Poynting vector)  $\vec{P} = [\vec{E}^{(s)}, \vec{H}^{(s)}]$ . The energy flux of a plane electromagnetic wave is equal to 0.5. Thus, the scattering cross section of the strip system considered is determined

by the formula:  $Q = \text{Im} \oint_{C_R} V_S^* \frac{\partial V_S}{\partial n} dl$ , where  $C_R$  is a circle with the radius  $R$ ;  $V_S = E_y^{(s)}$ ;  $n$  is a normal to the circle;  $dl$  is the differential of the arc-length. The radius  $R$  is large with respect to the geometrical parameters of the strip system. Thus, for  $V_S$  we can take the expression for the far zone field defined by the formula (7). Now we turn to the derivative in direction  $n$ :  $\frac{\partial V_S}{\partial n}$ . It can be presented in the

form  $\frac{\partial V_S}{\partial n} = \frac{\partial V_S}{\partial x} \cos \theta + \frac{\partial V_S}{\partial z} \sin \theta$ , where

$$V_S = -\frac{i}{4} \left[ \int_{-a_1}^{a_1} \varphi_1(t) H_0^{(1)}(kR_1) dt + \int_{-a_2}^{a_2} \varphi_2(t) H_0^{(1)}(kR_2) dt \right].$$

At first, we consider the partial derivatives:

$$\frac{\partial V_S}{\partial x} = \frac{ik}{4} \left[ \int_{-a_1}^{a_1} \varphi_1(t) H_1^{(1)}(kR_1) \left( \frac{x_1 - t}{R_1} \cos \alpha_1 + \frac{z_1}{R_1} \sin \alpha_1 \right) dt + \int_{-a_2}^{a_2} \varphi_2(t) H_1^{(1)}(kR_2) \left( \frac{x_2 - t}{R_2} \cos \alpha_2 - \frac{z_2}{R_2} \sin \alpha_2 \right) dt \right],$$

$$\frac{\partial V_S}{\partial z} = \frac{ik}{4} \left[ \int_{-a_1}^{a_1} \varphi_1(t) H_1^{(1)}(kR_1) \left( -\frac{x_1 - t}{R_1} \sin \alpha_1 + \frac{z_1}{R_1} \cos \alpha_1 \right) dt + \int_{-a_2}^{a_2} \varphi_2(t) H_1^{(1)}(kR_2) \left( \frac{x_2 - t}{R_2} \sin \alpha_2 + \frac{z_2}{R_2} \cos \alpha_2 \right) dt \right].$$

Substituting these expressions into  $\frac{\partial V_S}{\partial n}$ , we get the formula:

$$\frac{\partial V_S}{\partial n} = \frac{ik}{4} \times \left[ \int_{-a_1}^{a_1} \varphi_1(t) H_1^{(1)}(kR_1) \left( \frac{x_1 - t}{R_1} \cos(\theta + \alpha_1) + \frac{z_1}{R_1} \sin(\theta + \alpha_1) \right) dt + \int_{-a_2}^{a_2} \varphi_2(t) H_1^{(1)}(kR_2) \times \right.$$

$$\times \left[ \frac{x_2 - t}{R_2} \cos(\theta - \alpha_2) - \frac{z_2}{R_2} \sin(\theta - \alpha_2) \right] dt$$

Since  $R_m = r_m - t \cos \theta_m + O\left(\frac{1}{r_m}\right)$ , where  $r_m$ ,  $\theta_m$  are the local polar coordinates of a point at the circle, we have

$$H_1^{(1)}(kR_m) \approx -\frac{1+i}{\sqrt{\pi k r_m}} \exp[ik(r_m - t \cos \theta_m)],$$

$$\frac{x_m - t}{R_m} \approx \cos \theta_m.$$

Returning to the expression for  $\frac{\partial V_S}{\partial n}$ , we obtain the asymptotic expression

$$\begin{aligned} \frac{\partial V_S}{\partial n} &= \frac{-1+i}{4} \left[ \frac{e^{ikr_1}}{\sqrt{kr_1}} \hat{\phi}_1(\chi_1 \cos \theta_1) + \frac{e^{ikr_2}}{\sqrt{kr_2}} \hat{\phi}_2(\chi_2 \cos \theta_2) \right] = \\ &= \frac{-1+i}{4} \frac{e^{ikr_1}}{\sqrt{kr_1}} \left[ \hat{\phi}_{10} e^{-i\frac{k\rho}{2} \cos \theta} + \hat{\phi}_{20} e^{i\frac{k\rho}{2} \cos \theta} \right] + O(\chi^2). \end{aligned}$$

It is easy to see that the expression in the square brackets is the same as that given above for  $A(\theta)$  (7), so that the scattering cross section can be found using the formula:

$$\begin{aligned} Q &\approx \frac{1}{8k\pi} \int_0^{2\pi} \left| \phi_{10} e^{-i\frac{k\rho}{2} \cos \theta} + \phi_{20} e^{i\frac{k\rho}{2} \cos \theta} \right|^2 d\theta = \\ &= \frac{1}{4k} \left[ |\phi_{10}|^2 + |\phi_{20}|^2 + \right. \\ &\left. + 2(\operatorname{Re} \phi_{10} \operatorname{Re} \phi_{20} + \operatorname{Im} \phi_{10} \operatorname{Im} \phi_{20}) J_0(k\rho) \right]. \end{aligned}$$

In the case when one of strips is absent ( $\phi_{10} = 0$  or  $\phi_{20} = 0$ ), we have the expression:

$$Q = \frac{4a\pi^2}{\chi \left[ 4\ln^2 \frac{\chi\chi}{4} + \pi^2 \right]},$$

which is the same as for a single strip (cf. [4] and [1]).

The next electromagnetic characteristic of a scatterer is the scattering coefficient. It is defined as the ratio  $\sigma = Q/S$ , where  $S$  denotes the projection of the scatterer on the perpendicular line to the direction of the plane wave propagation.

For the two-segment system shown in Fig. 1 we have

$$S = 2(a_1 \sin(\theta_0 + \alpha_1) + a_2 \sin(\theta_0 + \alpha_2)),$$

where  $\theta_0$  lies in the interval

$$\left[ \arctan \frac{as}{\rho - ac}, \pi - \arctan \frac{as}{\rho + ac} \right],$$

here

$$as = a_1 \sin \alpha_1 + a_2 \sin \alpha_2,$$

$$ac = a_1 \cos \alpha_1 - a_2 \cos \alpha_2.$$

In Fig. 3 the numerical results are presented for the following frequency parameters:  $\chi_1 = 0.08$ ,

$$\chi_2 = 0.06, \quad k\rho = 0.18 \quad \left( \alpha_1 = \frac{\pi}{6}, \alpha_2 = \frac{\pi}{4} \right).$$

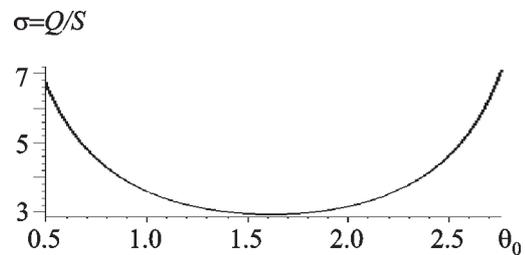


Fig. 3. Dependence of the scattering coefficient on angle  $\theta_0$

## 5. Conclusion

An asymptotic approach to investigation of the vectors of the electromagnetic field scattered by the system of strip is developed. It is based upon a solution of the systems of integral equations for the transverse and longitudinal current densities. The expressions for the main electromagnetic characteristics are deduced. The method described in this short paper has many advantages, one of which is its simplicity. Another feature is that it is quite general and can be used for arbitrary number of strips.

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## Електромагнітні характеристики простейшої системи полосок

**Г. И. Кошевой**

Исследованы основные электромагнитные характеристики простейшей некомпланарной системы узких полосок с параллельными краями. Они получены на основе метода интегральных уравнений с использованием локальных координатных систем. Приведены аналитические выражения для плотности продольных и поперечных поверхностных токов. Они были успешно проверены предельным переходом к одной ленте и позволяют исследовать как аналитически, так и численно взаимодействие лент. На основе этих выражений найдены асимптотические выражения для поля в дальней зоне, поперечного сечения рассеяния и коэффициента рассеяния. Полученные аналитические выражения достаточно просты и удобны для численных расчетов.

## Електромагнітні характеристики найпростішої системи стрічок

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Досліджено основні електромагнітні характеристики найпростішої некомпланарної системи вузьких стрічок з паралельними краями. Вони отримані на основі методу інтегральних рівнянь з використанням локальних координатних систем. Наведено аналітичні вирази для густини поздовжніх та поперечних поверхневих струмів. Вони були успішно перевірені граничним переходом до однієї стрічки і дозволяють дослідити як аналітично, так і числовими методами взаємодію стрічок. На основі цих виразів знайдено асимптотичні вирази для поля у дальній зоні, поперечного перерізу розсіювання та коефіцієнта розсіювання. Отримані аналітичні вирази є достатньо простими і зручними для числового обчислення.