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PROPAGATION OF SHORT PULSES IN PLASMA HALF-SPACE (IONOSPHERE)

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The exact solution of the problem in the half-space of non-homogeneous plasma is obtained, and an asymptotic method for the same task as well. The estimations of the non-linearity and the electron-atom collisions influence regarding the problem are carried out.

1. Introduction

This paper is the continuation of [1] where the rigorous solutions to the problem of short pulse propagation in plasma half-space for simplest models of the electrons density. Here, the asymptotic solution is presented enabling to approach the real distribution of electrons.

The 2-nd part of this work is on the peculiarities of taking into account the plasma electrons collisions with the atoms as short pulses are passing through.

2. The Asymptotic Solution

As a “large” parameter is proposed

$$k = \frac{L}{t_0 c},$$

where t_0 – the initial pulse duration at the medium boundary, L – the linear size of the pulse propagation region, c – the velocity of light.

In this case the field $u(z,t) = Z(z)T(t)$ obeys the equation

$$\frac{d^2 Z}{d\zeta} = k^2 p(\zeta) Z,$$

where $\zeta = z/L$, $p(\zeta) = t_0^2\left[\omega_p^2(\zeta) - \omega^2\right]$, ω_p – the plasma frequency, and the ordinary equation for $T(t)$.

If the field decreases as ζ increases relatively arbitrary cross-section ζ_0, we obtain [2]:

$$dZ = \frac{B_\omega d\omega}{\sqrt{k}\sqrt[4]{-p(\zeta)}} \exp\left(-k\int_{\zeta_0}^{\zeta} \sqrt{-p(\varsigma)}d\varsigma\right),$$
$$\zeta > \zeta_0, \quad \omega > \omega_p(\zeta),$$

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$$\zeta > \zeta_0, \quad \omega < \omega_p(\zeta).$$

If $z = 0$ at the plasma region boundary, then

$$p(0) = -\omega^2 t_0^2,$$

$$TdZ|_{\zeta=0} = \frac{B_\omega d\omega}{\sqrt{k}\sqrt[4]{\omega^2 t_0^2}} \exp(-i\omega t).$$

The incident onto the plasma field at $z = 0$ [1] is as

$$TdZ|_{\zeta=0} = \frac{E_{y0}}{4\pi} \frac{1 - 5\omega^2 t_0^2}{\left(\omega^2 t_0^2 + 0{,}25\right)^3}(1 + R)\exp(-i\omega t)d\omega,$$

where R – reflection coefficient. As these fields are equal as well as their corresponding normal derivatives and in view of $\frac{dZ}{dz} = \frac{1}{L}\frac{dZ}{d\zeta}$, one can obtain

$$\frac{1 + R_\omega}{1 - R_\omega} = \frac{ct_0}{L}\sqrt{\omega t_0}, \quad R_\omega = \frac{ct_0\sqrt{\omega t_0} - L}{ct_0\sqrt{\omega t_0} + L}.$$

Thence the frequency component of the Poynting vector of the reflected field is as

$$d\Pi = \frac{E_{y0}H_{0x}}{64\pi^2}\left[\frac{1 - 5\omega^2 t_0^2}{\left(\omega^2 t_0^2 + 0{,}25\right)^3}\right]^2 \left(\frac{ct_0\sqrt{\omega t_0} - L}{ct_0\sqrt{\omega t_0} + L}\right)^2 d\omega.$$

Thence it is easy to obtain Poynting vectors of the reflected and transmitted fields. In particular, the latter is like

$$\Pi = \frac{E_{y0}H_{0x}}{64\pi^2}\times$$
$$\int_0^\infty \left[\frac{1-5\omega^2 t_0^2}{\left(\omega^2 t_0^2+0{,}25\right)^3}\right]^2 \left[1-\left(\frac{ct_0\sqrt{\omega t_0}-L}{ct_0\sqrt{\omega t_0}+L}\right)^2\right] d\omega.$$

3. The Peculiarities of the Collisions

The non-linearity of the electron-atom (ion) collisions in the plasma excludes any making use of the canonic results for monochrome fields and radio impulses when the short pulses are dealt with.

When the magnetic field influence is ignored, the system of equations for the directed movement of the plasma electrons under the electric field is as in [3]:

$$\left.\begin{aligned} m\ddot{y} &= eE - m\nu\dot{y} \\ \frac{dT_e}{dt} &= \frac{2}{3\beta}e\dot{y}E - \delta\nu\left(T_e - T_p\right) \end{aligned}\right\},$$

where $\beta = 1.38\cdot 10^{-23}$ J/Kelvin – the Boltsmann constant, δ – the number of electron-molecule collisions per sec, T_e, T_p – temperatures of the electrons and plasma, m, e –electron's mass and charge.

Between the collisions the electron obeys the equation:

$$m\ddot{y} = eE;\quad \dot{y} = \frac{eE}{m}t + \dot{y}_0,$$

where $\dot{y}_0$ – velocity of electron after the previous collision. The work by the field between the collisions is as

$$A = \frac{me^2E^2\tau^2}{2} = \frac{me^2E^2}{2\nu_{\text{mean}}^2},$$

where τ – the mean time span between collisions.

As a result of a recurrent collision the electron looses the part of its energy specified by the multiplier $2m/M$, where m, M – the masses of electron and the molecule. In this way the kinetic energy of the directed move due to the 1-st collision is equal to $\frac{m\dot{y}_0^2}{2}\left(1-2\frac{m}{M}\right)$, which replies the new velocity $\dot{y}_1 = \dot{y}_{\mathbf{0}}\sqrt{1-2\frac{m}{M}}$. As this process continues, we obtain

$$\dot{y}_n = \dot{y}_{\mathbf{0}}\,\frac{eE}{m}\sum_{i=1}^{n}\frac{\left(\sqrt{1-{}^{2m}\!/\!_{M}}\right)^i}{\nu_i}.$$

Here it is allowed for ν_i to vary in-between collisions depending on the electron temperature variation. Thus for the electron-atom and the electron-molecule collisions.

$$\nu_{\text{ef}} = \nu\sqrt{{}^{T_e}\!/\!_{T_p}},$$

where ν_{ef} – the effective number of collisions at $T_e = T_p$.

One can get convinced that in the ionosphere conditions, at the achievable short pulse power levels as the ionosphere is reached, and the number of collisions is small enough, one obtains $\sqrt{{}^{T_e}\!/\!_{T_p}} \approx 1$. Therefore

$$\dot{y}_n = \dot{y}_{\mathbf{0}} + \frac{eE}{\nu_{m0}}\sum_{i=1}^{n}\left(\sqrt{1-2m/M}\right)^i =$$
$$\dot{y}_{\mathbf{0}} + \frac{e}{m}\frac{E}{\nu_{m0}}\frac{1-\left(\sqrt{1-2m/M}\right)^n}{1-\sqrt{1-2m/M}}.$$

When the pulse duration is less than the collision-to-collision time span, the initial system of equations is as

$$\left.\begin{aligned} m\ddot{y} &= eE, \\ \frac{dT_e}{dt} &= \frac{2}{3\beta}e\dot{y}E \end{aligned}\right\}\quad \text{for}\quad T_e|_{t=0} = T_p.$$

Thence $T_e|_{t=0} = T + \frac{e^2E^2t_0^2}{3\beta m}$.

After a short enough pulse has left, the equations determining the dynamics of leveling of the temperatures of the electrons and plasma are valid:

$$\frac{dT_e}{dt} = -\delta\nu\left(T_e - T_p\right),$$
$$T_e = T_p + \frac{e^2E^2t_0^2}{3\beta m}\exp[-\delta\nu(t-\tau)],$$
$$\dot{y} = \dot{y}(\tau)\exp[-\nu(t-\tau)].$$

In the ionosphere, for layers “E” and “F” correspondingly

$$\tau_{\text{E}} = \frac{1}{\nu_{\text{E}}} \approx 10^{-5}\ \text{s},\quad \tau_{\text{F}} = \frac{1}{\nu_{\text{F}}} \approx 10^{-2}\ \text{s}.$$

Consequently, the pulses with duration not greater than 10^{-6} s should have passed through without any collision effect at all.

References

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РАСПРОСТРАНЕНИЕ КОРОТКИХ ИМПУЛЬСОВ В ПЛАЗМЕННОМ ПОЛУПРОСТРАНСТВЕ (ИОНОСФЕРЕ)

А.Л. Гутман

Получены точное и асимптотическое решения задачи распространения коротких импульсов в полубесконечном пространстве с неоднородной плазмой. Дана оценка влияния в этой задаче нелинейности и столкновений електрон-атом.

РОЗПОВСЮДЖЕННЯ КОРОТКИХ ІМПУЛЬСІВ У ПЛАЗМОВОМУ ПІВПРОСТОРІ (ІОНОСФЕРІ)

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Отримано точний та асимптотичний розв'язки задачі розповсюдження коротких імпульсів у півнескінченному просторі з неоднорідною плазмою. Дано оцінку впливу у цій задачі нелінійності та зіткнень електрон-атом.