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# ULTRA-SHORT PULSE EXCITATION IN A SEMI-INFINITE WAVEGUIDE BY A RELATIVISTIC ELECTRON BUNCH

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The theoretical investigations and simulation are performed for the emission of a relativistic electron bunch during the injection into a semi-infinite vacuum/dielectric waveguide. The exact solution of the problem is obtained for the first time. Contrary to the previously used saddle-point technique, we applied a number of successive conform transformations of integration area in order to carry out the inverse Fourier transformation. The power and the frequency spectrum of high intensity pulse of ultra-wideband transition radiation, excited by a finite size electron bunch in a vacuum waveguide, are calculated numerically. It is shown that in a dielectric waveguide the short pulse of Cherenkov wake field drifts behind the relativistic bunch with the group velocity.

## 1. Introduction

At present, much attention is focused on the problem of generation of short intense electromagnetic pulses whose frequency spectra width is comparable with their mean frequency, the so-called "ultra-wideband" (UWB) pulses [1, 2]. The current interest to the problem of pulsed emission of high-power electromagnetic signals arises primarily due to their application in UWB radiolocation.

Along with traditional methods of generation of short UWB pulses based on the use of UWB antennas (TEM-horns, spiral and biconical antennas, and others [3]) the intense pulsed relativistic electron beams (IREBs) can be used. Short (and ultra-short) IREBs with durations of 1–100 ps, energies of 0.5–1 MeV, and peak currents of 1–100 kA can be either produced by transformation of a continuous IREB into a sequence of electron pulses (modulated beams) [4] or directly generated in high-current devices (explosive-emission diodes).

For the most efficient generation of electromagnetic UWB pulses the non-resonant (impact) mechanisms of IREBs irradiation should be used, such as charging of a rod antenna by REB [5] or impact excitation of TEM-horn antenna by IREB [6]. High-power UWB pulses can be generated not only in direct processes of the excitation of UWB antennas by IREBs but also due to the effect of coherent transition radiation [7]. The transition radiation from an individual charged particle is one of the fundamental elementary radiation processes [8]. This effect occurs when a charged particle moves through the electrically non-uniform medium. The transition radiation effect can be greatly enhanced by using small dense bunches of charged particles [9]. The radiation from a bunch consisting of $N$ particles is $N^2$ times more intense than that from one particle. Because of this effect, electron bunches provide efficient means of intense electromagnetic transition radiation generation.

Here we theoretically investigate the excitation of UWB transition radiation during the injection of a short-pulsed IREB into a semi-infinite circular cross-section waveguide whose entrance (through which the beam is injected) is short-circuited by a conducting diaphragm. In such a waveguide, the large dispersion of electromagnetic waves is the pecularity of the transition radiation, so that the shape of a UWB pulse of transition radiation propagating in a waveguide will deform permanently.

In the case of dielectric filling the intense Cherenkov wake field excitation can take place. It can be applied for wake-field acceleration of charged particles [10-11] or for radiation sources [12,13]. In the case of short-pulsed IREB the interference of big number of radial modes leads to the essential peaking of the wake field with formation of narrow spikes of alternative sign [14]. The spatial structure of excited field is determined by the spatially limited Cherenkov field and transition radiation field.

## 2. Non-Resonant Wideband Emission in a Semi-Infinite Waveguide

We consider a semi-infinite $(0 \le z \le \infty)$ cylindrical metal waveguide of radius $b$ which is filled with a homogeneous dielectric with permittivity $\varepsilon$. The waveguide input end ($z = 0$) is short-circuited by a metal wall transparent to relativistic electrons. An axisymmetric monoenergetic electron bunch is injected through the metal wall and moves with a constant velocity $v_0 < c/\sqrt{\varepsilon}$ along the symmetry axis of the waveguide (the $z$ axis). We start with the determination of the field of an infinitely short and infinitely thin charged ring with the charge density

$$\rho = \frac{-eN}{2\pi r_0 v_0}\delta(r - r_0)\delta\left(t - t_0 - \frac{z}{v_0}\right), \qquad (1)$$

where $-e$ is the charge of an electron, $N$ is the number of electrons in the ring, $v_0$ is the ring velocity, $r_0$ is the ring radius, and $t_0$ is the time at which the ring enters the waveguide. Solving Fourier-transformed Maxwell's equations with allowance for the boundary condition $E_r = 0$ at the end metal wall, we obtain the following expression for the radial electric field of an axisymmetric E-wave:

$$E_r(t,r,z,t_0,r_0) = \frac{2Ne}{\pi b \varepsilon v_0} \times \sum_n \frac{\omega_{0n}^2 J_0(\lambda_n r_0/b) J_1(\lambda_n r/b)}{\lambda_n J_1^2(\lambda_n)}\{I_{2n} - I_{1n}\}, \qquad (2)$$

$$I_{1n} = \int_{-\infty}^{\infty} d\omega \frac{\exp[-i\omega t + i\omega(t_0 + z/v_0)]}{(\omega - i\omega_{0n})(\omega + i\omega_{0n})}, \qquad (3)$$

$$I_{2n} = \int_{-\infty}^{\infty} d\omega \frac{\exp\left[-i\omega\tau + i\xi\sqrt{\omega^2 - \alpha_n^2}\right]}{(\omega - i\omega_{0n})(\omega + i\omega_{0n})}, \qquad (4)$$

where $\tau = t - t_0$, $\xi = z\sqrt{\varepsilon}/c$, $\alpha_n = \lambda_n c/(b\sqrt{\varepsilon})$, $\omega_{0n} = \lambda_n v_0 / \left(b\sqrt{1 - \varepsilon v_0^2/c^2}\right)$, and $\lambda_n$ is the $n$-th root of the Bessel function $J_0$.

Integral (3) describes the Coulomb field of a charge moving in an infinite waveguide. Integral (4) corresponds to free oscillations of a cylindrical waveguide and describes the transition radiation. The exact analytic solution of integral similar to (4) was obtained by Denisov [15] in studying the propagation of an electromagnetic signal in an ionized gas. Burshtein and Voskresenskij in [16] used the saddle point technique in order to obtain the approximate solution of integral (4) under condition of Cherenkov resonance. Below we applied the method proposed by Denisov.

In order to calculate the integral (4) we used a sequence of substitutions: $p = -i\omega$, $\zeta = \left(\sqrt{p^2 + \alpha_n^2} - p\right)/\alpha_n$, and $w = -\zeta/\beta$, where $\beta = \sqrt{(\tau - \xi)/(\tau + \xi)}$. After these conform transformations we passed from the integration along the real axis to the integration along the closed circular contour. This allows us to separate the integral form of the Bessel functions. Finally [17] we obtain the expression for the total electric field (2) of a thin annular electron bunch (1) in the form of the superposition of the Coulomb field of a moving charge and the transition radiation field:

$$E_r(t,r,z,t_0,r_0) = E_r^{coul}(t,r,z,t_0,r_0) + E_r^{trans}(t,r,z,t_0,r_0), \qquad (5)$$

$$E_r^{coul}(t,r,z,t_0,r_0) = -\frac{2Ne}{b^2\varepsilon\sqrt{1 - \varepsilon v_0^2/c^2}} \times \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r_0/b) J_1(\lambda_n r/b)}{J_1^2(\lambda_n)} \times \{\vartheta_1 \exp[\omega_{0n}(t - t_0 - z/v_0)] + \vartheta_2 \exp[-\omega_{0n}(t - t_0 - z/v_0)]\}, \qquad (6)$$

$$E_r^{trans}(t,r,z,t_0,r_0) = \frac{4Ne}{b^2\varepsilon\sqrt{1 - \varepsilon v_0^2/c^2}} \times \sum_n \frac{J_0(\lambda_n r_0/b) J_1(\lambda_n r/b)}{J_1^2(\lambda_n)} \times \left\{\vartheta_1\left[\sum_{m=0}^{\infty}\left(q_1^{2m+1} - q_2^{2m+1}\right) J_{2m+1}(y_n) + \frac{1}{2}\exp[\omega_{0n}(t - t_0 - z/v_0)]\right] + \vartheta_2\left[\sum_{m=0}^{\infty}\left(q_1^{2m+1} + q_2^{-2m-1}\right) J_{2m+1}(y_n) + \frac{1}{2}\exp[-\omega_{0n}(t - t_0 - z/v_0)]\right]\right\}, \qquad (7)$$

where $\vartheta_1 = 1$ if $z_0 \le z < z_{pr}$, else $\vartheta_1 = 0$; $\vartheta_2 = 1$ if $0 \le z < z_0$, else $\vartheta_2 = 0$; $z_0 = (t - t_0)v_0$ is the position of the ring-shaped bunch (1), $z_{pr} = (t - t_0)v_{pr}$ is the position of the precursor of transition field, $v_{pr} = c/\sqrt{\varepsilon}$ is the maximal velocity of the EM perturbation propagation in the dielectric waveguide;

$$q_{1,2} = \sqrt{\frac{(t - t_0 - z\sqrt{\varepsilon}/c)(c \mp \sqrt{\varepsilon}v_0)}{(t - t_0 + z\sqrt{\varepsilon}/c)(c \pm \sqrt{\varepsilon}v_0)}},$$

$$y_n = \frac{\lambda_n c}{b\sqrt{\varepsilon}}\sqrt{(t - t_0)^2 - \frac{z^2\varepsilon}{c^2}}.$$

The quasistatic field component (6) and the transition radiation field component (7) are defined (and are nonzero) in the region $z < z_{pr}$. For $t > t_0$, neither Coulomb nor transition radiation fields enter the region to the right of the point $z = z_{pr}$. The fastest and the highest-frequency component of the elec-

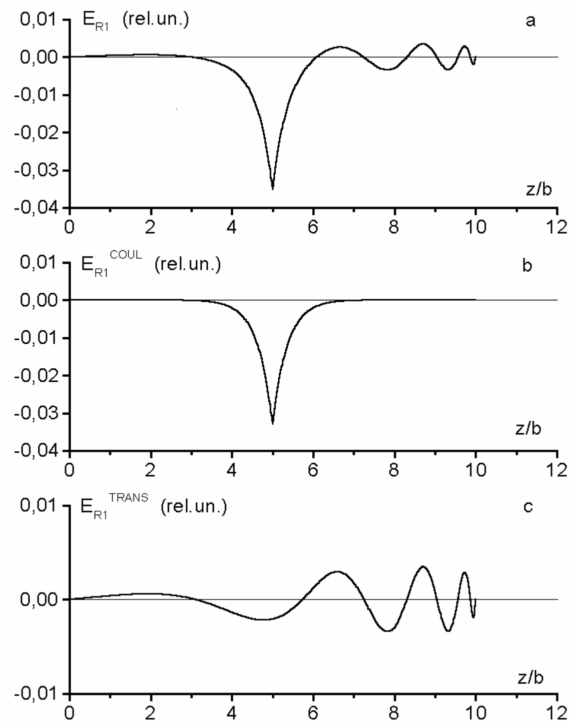


**Fig. 1.** *The first harmonic: a – total field, b – coulomb field, c – transition field.* $tc/b = 10$, $r/b = 1$, $v_0/c = 0.5$, $\varepsilon = 1$

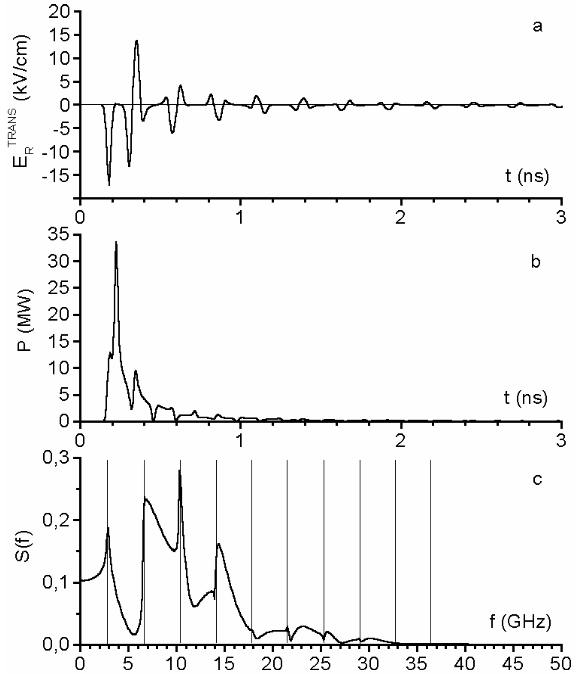


**Fig. 2.** *The characteristics of the transition radiation: a – field, b – power, c – spectrum.* $z = 4$ cm, $r = 1$ cm, $b = 4$ cm, $\varepsilon = 1$, $v_0/c = 0.9$, $L_b = 2$ cm, $a = 0.5$ cm

tromagnetic signal is the precursor, which propagates with the velocity $v_{pr}$ [18]. Since the bunch propagation velocity is $v_0 < v_{pr}$, the field overtakes the bunch. A qualitative pattern of the propagation of a transition radiation pulse is illustrated in Fig. 1, which shows the longitudinal profiles of the first harmonic of the total radial electric field and its Coulomb and transition radiation components. The bunch generates a UWB transition radiation pulse, whose shortest wavelength components are the components of the precursor. The oscillation amplitude in the pulse decreases toward the precursor, so that the total field vanishes in the cross section $z = z_{pr}$.

For simulations we chose an electron bunch with the following current density distribution:

$$j_z(r_0, t_0) = j_0 \cdot J_0\left(\lambda_1 \frac{r_0}{a}\right) \exp\left[-4\left(\frac{2t_0}{T_b} - 1\right)^2\right], \quad (8)$$

where $j_0$ is the peak current density, $0 < t_0 < T_b$, $T_b$ is the bunch duration, $r_0 < a$, $a$ is the bunch radius.

The characteristics of the transition radiation signals near the waveguide input end are illustrated in Fig. 2. We can see that an electromagnetic pulse near the waveguide input end is characterized by a large field amplitude (15 kV/cm), high maximum power (about 33 MW), and short duration (less than 1 ns). The spectrum of this signal is broadband and has sharp narrow peaks at frequencies close to the critical frequencies $f_n = \lambda_n c/(2\pi b\sqrt{\varepsilon})$ of the waveguide. The presence of the low-frequency $(f < f_1)$ part of the spectrum can be explained by the fact that the transition radiation signal has propagated only a short distance and, therefore, did not have enough time to form completely.

## 3. Wake-Field Excitation in a Semi-Infinite Dielectric Waveguide

In the case of Cherenkov resonance $(v_0 > c/\sqrt{\varepsilon})$ the field, excited by the thin charged ring (1) can be written [19], similar to [16], in the form of superposition of spatially limited Cherenkov radiation field and transition radiation field:

$$E_z(t, r, z, t_0, r_0) = E_z^{cher}(t, r, z, t_0, r_0) + E_z^{trans}(t, r, z, t_0, r_0), \quad (9)$$

$$E_z^{cher}(t, r, z, t_0, r_0) = \frac{4Ne}{b^2\varepsilon} \sum_n \frac{J_0(\lambda_n r_0/b) J_0(\lambda_n r/b)}{J_1^2(\lambda_n)} \times \vartheta(z, z_{gr}, z_0) \cos[\tilde{\omega}_{0n}(t - t_0 - z/v_0)], \quad (10)$$

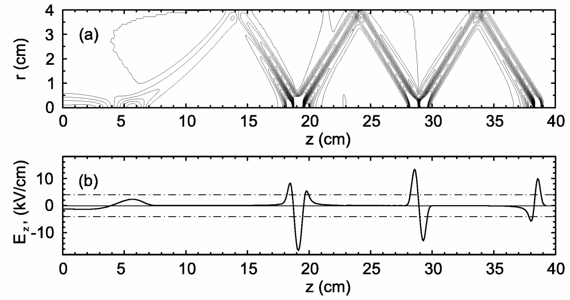


**Fig. 3.** *The topography of the field $E_z$ in the semi-infinite waveguide in the case of Cherenkov resonance.*
*(a) – the level curves of the field $E_z$,*
*(b) – the respective dependence $E_z$ vs. $z$ obtained at $r = 0$.*
*Level curves of field are drawn with a step of 0.4 kV/cm in the range from – 4 kV/cm to 4 kV/cm. Dash-dot lines in figure (b) mark the limits of this range.*
$tc/b = 10$, $b = 4$ *cm*, $\varepsilon = 2.6$, $Lb = 1$ *cm*,
$a = 0.5$ *cm*, $v_0/c = 0.9798$, $Q_b = 1.6$ *nC*

$$E_z^{trans}(t,r,z,t_0,r_0) = \frac{4Ne}{b^2\varepsilon}\sum_n \frac{J_0(\lambda_n r_0/b)J_0(\lambda_n r/b)}{J_1^2(\lambda_n)} \times \Big\{\vartheta(z,z_{gr},z_{pr})\sum_{m=1}^{\infty}(-1)^m(\tilde{q}_1^{2m} - \tilde{q}_2^{2m})J_{2m}(y_n) + \vartheta(z,0,z_{gr})\big[J_0(y_n) + \sum_{m=1}^{\infty}(-1)^m(\tilde{q}_1^{2m} + \tilde{q}_2^{-2m})J_{2m}(y_n)\big]\Big\} \quad (11)$$

$\vartheta(z,z_1,z_2) = 1$ if $z_1 \le z < z_2$, else $\vartheta(z,z_1,z_2) = 0$; $z_{gr} = (t - t_0)v_{gr}$ is the position of the group wavefront, $v_{gr} = c^2/\varepsilon v_0$ is the group velocity of the resonance wave; $\tilde{\omega}_{0n}^2 = -\omega_{0n}^2$, $\tilde{q}_{1,2}^2 = -q_{1,2}^2$.

In Fig. 3(a) with the help of level curves the 2D (in the plane $z - r$) picture of distribution of longitudinal electric field excited by relativistic electron bunch with density (8) is represented. The bunch sizes are small in comparison with those of waveguide, so we can think that position of "group wavefront" is $z_{gr} = 15.6$ cm, coordinate of precursor is $z_{pr} = 24.8$ cm, and bunch coordinate is $z_0 = 38.7$ cm. In the region $z_{pr} < z < z_0$ the intense Cherenkov wake wave exists. Structure of this wave is formed as a result of periodic reflections of Cherenkov cone from the sidewalls of waveguide. In the region $0 < z < z_{pr}$ the transition radiation field superimposes on the Cherenkov field. Weak transition oscillations in the precursor region $20\text{ cm} < z < 23\text{ cm}$ still can be noticed against the intense Cherenkov field. Behind $z_{gr}$ the field is small and its structure is different from the one of Cherenkov wave. Note the high amplitude and small width of field spikes in Fig. 3(b). $E_z$ is maximal at the waveguide's axis, where the waves, reflected from sidewalls, are focusing.

## 4. Conclusion

When a charged bunch enters a semi-infinite cylindrical waveguide, it generates a transition radiation. If the Cherenkov resonance condition is not satisfied, the excited electromagnetic field is a superposition of the quasistatic field of a moving charge and the transition radiation field. The fastest component of the field (the precursor) propagates with the velocity $c/\sqrt{\varepsilon}$, which is higher than the bunch velocity.

The spectrum of the transition radiation signal is broadband and contains peaks corresponding to several radial modes. The wide and asymmetric peaks that are clearly distinguished in the spectrum occur at frequencies somewhat higher than the corresponding critical frequencies of the waveguide. The transition radiation signal calculated near the waveguide entrance is characterized by high maximum power and short duration.

If the Cherenkov resonance condition is satisfied, the excited field consists of spatially limited Cherenkov radiation field and transition radiation field. Accounting of the boundary leads to the appearing of the effect of wake field's drift after the leading bunch with the group velocity of resonance wave. This results in limitation of intense wake field region in the longitudinal direction.

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## ВОЗБУЖДЕНИЕ УЛЬТРАКОРОТКОГО ИМПУЛЬСА В ПОЛУОГРАНИЧЕННОМ ВОЛНОВОДЕ РЕЛЯТИВИСТСКИМ ЭЛЕКТРОННЫМ СГУСТКОМ

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Выполнены теоретические исследования и численное моделирование излучения релятивистского электронного сгустка или последовательности сгустков при инжекции в полуограниченный вакуумный или диэлектрический волновод. Получено точное решение задачи. Вместо использовавшегося ранее метода седловой точки для нахождения асимптотического решения мы воспользовались несколькими конформными отображениями области интегрирования, чтобы выполнить обратное преобразование Фурье. С помощью численного моделирования определены мощность и частотный спектр интенсивного импульса сверхширокополосного переходного излучения, возбуждаемого электронным сгустком конечных размеров в вакуумном волноводе. Показано, что в диэлектрическом волноводе возбуждается короткий импульс Черенковского кильватерного поля, распространяющийся за сгустком с групповой скоростью.

## ЗБУДЖЕННЯ УЛЬТРАКОРОТКОГО ІМПУЛЬСУ В НАПІВОБМЕЖЕНОМУ ХВИЛЕВОДІ РЕЛЯТИВИСТСЬКИМ ЕЛЕКТРОННИМ ЗГУСТКОМ

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Виконані теоретичні дослідження та чисельне моделювання випромінювання релятивістського електронного згустка або послідовності згустків при інжекції в напівобмежений вакуумний або діелектричний хвилевод. Отримане точне рішення задачі. Замість використаного раніше методу сідлової точки для знаходження асимптотичного рішення ми використали декілька конформних відображень області інтегрування, щоб зробити зворотне Фур'є-перетворення. З використанням чисельного моделювання знайдена потужність і частотний спектр інтенсивного імпульсу надширокосмугового перехідного випромінювання, що збуджується електронним згустком скінченних розмірів в вакуумному хвилеводі. Показано, що в діелектричному хвилеводі збуджується короткий імпульс Черенківського кільватерного поля, що розповсюджується за згустком з груповою швидкістю.