Radio Physics and Radio Astronomy, 2002, v. 7, No. 4, pp. 459-461

MODIFICATION OF THE GELFAND-LEVITAN METHOD FOR 1-D MULTYLAERED STRUCTURE INVERSE PROBLEM

O. Drobakhin, M. Andreev, A. Novomlinov, V. Korotkaya, A. Sazonov

Radiophysics Dept., Dniepropetrovsk State University,
13 Nauchny str., 49050, Dniepropetrovsk, Ukraine

For dielectric slab with step profile of dielectric constant the Gelfand-Levitan method is correct if peaks of time-domain reflected signal are close to δ-pulses. Combination of parametric spectral methods for obtaining time-domain signal from frequency domain data and Gelfand-Levitan method for time-domain signal processing can help to improve the solution of the problem. Results of numerical simulation are presented.

1. Introduction

1-D inverse problem for multylayered dielectric structures is a fundamental problem of physics. The profile of dielectric constant as function of distance has to be reconstructed. A well-known approach to the solution of the problem is the Gelfand-Levitan method [1]. The main advantage of this approach is capability of discrete (step) profile recovering with taking into account multiple reflections.

2. Basic Relations

The Gelfand-Levitan method can be applied to discrete multylayered dielectric structures with step profile of dielectric permittivity. It can be used successfully if reflectogramma of structure is obtained even for situation with multiple reflections in every layer but only for ultrashort time pulses.

Let us consider a multylayered dielectric structure which consists of n layers with $\varepsilon_1, \varepsilon_2 \ldots \varepsilon_n$ and thicknesses $d_1, d_2 \ldots d_n$; ε_0 is dielectric constant of air.

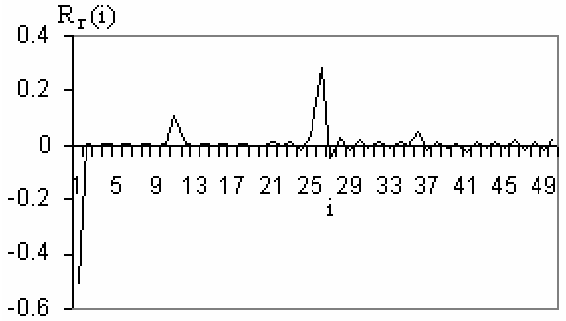

Fig. 1. *Time-domain reflection coefficient for two-layered structure calculated using discrete Fourier transform* ($f_1 = 0.499$ *GHz,* $N = 50$, $d_1 = 1$, $d_2 = 2$ *cm)*

The essence of the Gelfand-Levitan method consists in serial exception of the upper most layer influence in relation to direction of the falling wave. For this purpose the linear equation system is made reflected wave data R_i with unknown function $K(k, 2j-k)$ for system consisting of the k top layers.

$$\left\{\begin{pmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & \ldots & R_0 \\ 0 & R_0 & \ldots & R_1 \\ \vdots & \vdots & \ddots & \vdots \\ R_0 & R_1 & \ldots & R_{k-1} \end{pmatrix}\right\} \cdot \begin{pmatrix} K(k,2-k) \\ K(k,4-k) \\ \vdots \\ K(k,k) \end{pmatrix} = \begin{pmatrix} -R_0 \\ -R_1 \\ \vdots \\ -R_{k-1} \end{pmatrix}. \quad (1)$$

After solving this system of equations, we will find value of $K(k,k)$ and then after substitution of this value to the formula

$$r_{k-1} = 1 \Big/ \left\{ \prod_{i=0}^{k-2} (1 - r_i^2)[1 + K(k,k)] \right\} - 1 , \quad (2)$$

we will find the reflectivity factor of the $(k-1)$ layer.

The restoration of dielectric permeability is described by the formula:

$$\varepsilon_i = \varepsilon_{i-1} \left((1 - r_{i-1}) / (1 + r_{i+1}) \right)^2 . \quad (3)$$

The Gelfand-Levitan method has a disadvantage for situation of peaks with non-similar to δ -peak

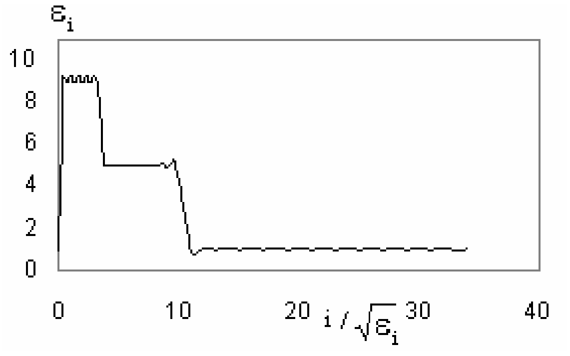

Fig. 2. *Profile of dielectric constant recovered by Gelfand-Levitan method for signal of Fig. 1*

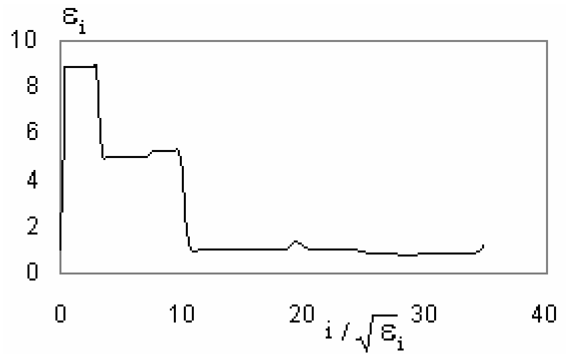

Fig. 3. *Profile of dielectric constant recovered by Gelfand-Levitan method for data received by generalized pencil of matrix method*

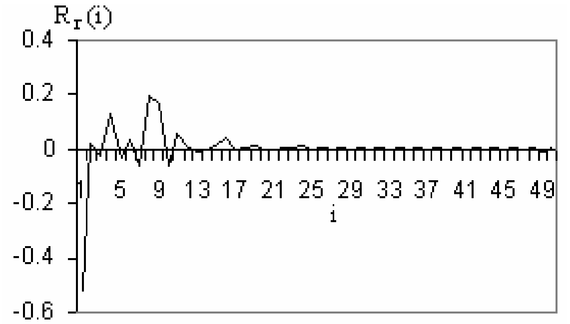

Fig. 4. *Time-domain reflection coefficient for two-layered structure calculated using discrete Fourier transform ($f_1 = 0.15$ GHz, $N = 50$, $d_1 = 1$, $d_2 = 2$ cm)*

form. The accuracy of the profile reconstruction is being degraded. For finite frequency band signals there are ripples in reconstructed profile of dielectric constant. This situation is rather usual for microwave measurements.

Parametric spectral methods such as the Prony method, the generalized pencil of matrix method [2] and the maximum of likelihood method [3] can be applied instead of discrete Fourier transform for obtaining time-domain signal. There is a simple approach [4] for data interpretation but the approach is correct in case of negligibly low level of reverberations. Applications of parametric spectral analysis methods allows one to avoid disadvantages of traditional application of the Gelfand-Levitan method to time-domain signal obtained by discrete Fourier transform.

3. Numerical Simulation

For numerical simulation of the Gelfand-Levitan method using different methods of parametric spectral analysis the time-domain reflection coefficient has been synthesized on the base of frequency-domain data calculated under suggestion of planar-wave approximation. The parameters of two-layered structure were $\varepsilon_1 = 9$ and $\varepsilon_2 = 5$. The thicknesses were $d_1 = 1$ cm and $d_2 = 2$ cm. The values of step in frequency domain were chosen equal to $\Delta f_1 = 0.499$ GHz, $\Delta f_2 = 0.15$ GHz. The number of frequencies N was 50. Thus the frequency band were 24.45 and 7.35 GHz correspondingly.

For data presented in fig. 2, 3, 5, 6 one sample of x-axis corresponds to geometrical sample equal to $\frac{1}{2\pi N \Delta f_i}$ with additional multiplier $1/\sqrt{\varepsilon_i}$. The value ε_i was chosen equal to value in this point.

Time discrete for data in Fig. 1 were chosen in the manner provided time-domain peak form closely similar to form of δ -pulses.

Fig. 2 confirms Gelfand-Levitan method capability of correct reconstruction of dielectric constant profile for case of $\sin x / x$ peaks which are closely similar to δ -pulses. Non-sufficient ripples in the reconstructed profile are occurred due to side-lobes of functions of $\sin x / x$. Results of dielectric constant profile reconstruction by the Gelfand-Levitan method for time-domain signals obtained by generalized pencil of matrix method is presented in Fig. 3.

The Prony method and the maximum of likelihood method give similar. It is clear that application of the three methods of parametric spectral analysis allows one to reconstruct profile more correctly. The parasitic ripples have been vanished. For some parts of the profile the first approach provides more accurate results but this situation occurred only for specific situation of δ -form of pulses. In practical situation δ -form of pulses can be occurred seldom.

Results for case $\Delta f_2 = 0.15$ GHz are presented in Fig. 4-6. After discrete Fourier transform obtained peaks in time domain have form non-similar to δ -pulses. Recovering the profile by direct application of the Gelfand-Levitan method implies the appearance of ripples with large amplitudes comparable with values of the dielectric constant. These oscilla-

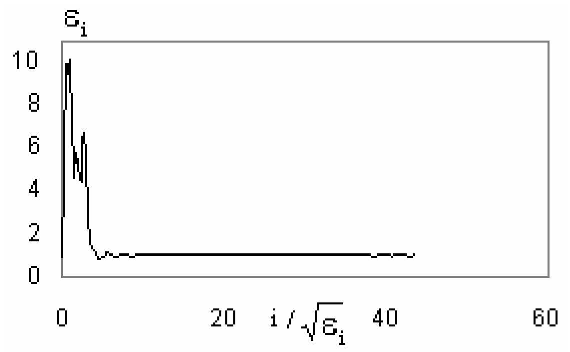

Fig. 5. *Profile of dielectric constant recovered by the Gelfand-Levitan method for signal of Fig. 4*

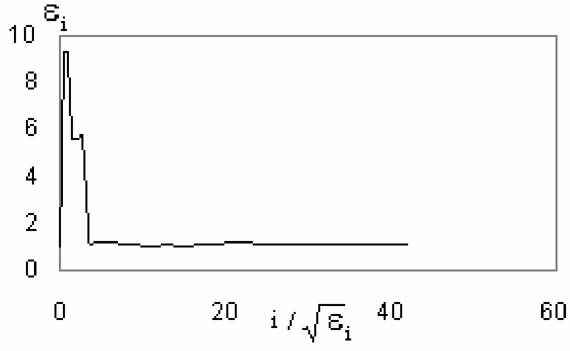

Fig. 6. *Profile of dielectric constant recovered by Gelfand-Levitan method for data received by generalized pencil of matrix method*

tions managed to be avoided by use of the spectral parametrical analysis methods.

Results of profile reconstruction by the Gelfand-Levitan method for time-domain data obtained from the same data of Fig. 4 using parametric spectral methods are presented in Fig. 6. Restored dielectric permeability structure due to use of this methods becomes more smooth and oscillations disappear.

Worse result was obtained for case of the Prony's method but even for this situation the ripples were disappeared. The best results were obtained for maximum likelihood method and generalized pencil of matrix method. The approaches provide rather high accuracy of estimation.

4. Conclusions

The results of numerical experiments allow to make conclusions that the Gelfand-Levitan method is the most simple in realization if time characteristic of the wave reflected from the layered structure are obtained by discrete Fourier transform from data obtained experimentally or numerically in frequency domain. This way is more economic concerning computing expenses but it works adequately when the width of frequency band is chosen in the manner that peaks have δ-pulse form and non-overlapped. In practice it is difficult enough for realizing if layers have different electric thicknesses. The spectral parametrical analysis methods used allow one to solve the problem.

References

1. K. Aki, P.G. Richards. Quantitative seismology. Theory and methods. W.H. Freeman and Company, 1980.
2. Y. Hua, T.K. Sarkar. Generalized Pencil-of-Function Method for Extracting Poles of an EM System from Its Transient Response. IEEE Trans. Antennas and Propag, **AP-37**, No. 2, pp. 229-233, Feb. 1989.
3. H. Vanhamme. High resolution frequency-domain reflectometry. IEEE Trans. Instrumentation and Measurement, **39**, No. 2, pp. 369-375, Apr.1990.
4. M.V. Andreev, V.F. Borulko, O.O. Drobakhin. One-dimensional Inverse Problem Solution for Multilayered Dielectric Structures Using Least-Square Spectral Estimation Method. Proc. of the 1995 URSI Int. Symp. on Electromagnetic Theory, St.Petersburg, Russia, May 23-26, 1995, pp. 148-151.

МОДИФИКАЦИЯ МЕТОДА ГЕЛЬФАНДА-ЛЕВИТАНА ДЛЯ ОБРАТНОЙ ЗАДАЧИ ОДНОМЕРНОЙ МНОГОСЛОЙНОЙ СТРУКТУРЫ

О. Дробахин, М. Авдеев, А. Новомлинов, В. Короткая, А. Сазонов

Для диэлектрической плиты со ступенчатым профилем диэлектрической постоянной применим метод Гельфанда-Левитана, если пики отраженного сигнала близки к δ-импульсам. Комбинация параметрических спектральных методов для получения сигнала во временной области по данным из частотной области и метод Гельфанда-Левитана для обработки сигнала во временной области позволяют получить усовершенствованный алгоритм решения задачи. Приведены результаты численного моделирования.

МОДИФІКАЦІЯ МЕТОДУ ГЕЛЬФАНДА-ЛЕВІТАНА ДЛЯ ЗВОРОТНОЇ ЗАДАЧІ ОДНОВИМІРНОЇ БАГАТОШАРОВОЇ СТРУКТУРИ

О. Дробахін, М. Авдєєв, О. Новомлінов, В. Короткая, А. Сазонов

Для діелектричної плити зі східчастим профілем діелектричної сталої метод Гельфанда-Левітана застосовний, якщо піки відбитого сигналу близькі до δ-імпульсів. Комбінація параметричних спектральних методів для отримання сигналу в часовій області та метод Гельфанда-Левітана для обробки сигналу в часовій області дозволяють отримати удосконалений алгоритм розв'язання задачі. Наведено результати чисельного моделювання.