Radio Physics and Radio Astronomy, 2002, v. 7, No. 4, pp. 471-474

WAVELET ANALYSIS AND ULTRA WIDEBAND SIGNALS

L.F. Chernogor, O.V. Lazorenko, S.V. Lazorenko

Kharkiv V. N. Karazin National University
4, Svoboda Square, 61077, Kharkov, Ukraine
E-mail: Leonid.F.Chernogor@univer.kharkov.ua

The wavelet analysis is offered to be used for the description of the ultra wideband signals. The expediency of such approach is justified. It is displayed, that wavelets are ultra wideband signals. The wideband indexes for most frequently used wavelets are evaluated. The advantages of the wavelet analysis before conventional methods are considered at research of ultra wideband signals. As an example the wavelet spectra of some model ultra wideband signals are considered.

1. Introduction

For the last years the advancement of science of engineering has resulted in appearance of recent trends, one of which is the application of non-traditional kinds of signals in their different areas, in particular, in radiolocation and communications. To such new kinds of signals belong ultra wideband (UWB) signals possessing a number of advantages in comparison with widely used narrow-band and wide-band signals.

It is accepted to that a UWB signal is a signal, the wideband index μ of which obeys to a condition $1 \le \mu < 2$, where

$$\mu = \frac{\Delta f}{f_0} = 2\frac{f_{\max} - f_{\min}}{f_{\max} + f_{\min}}, \qquad (1)$$

Δf, f_0, $f_{\min}$ and $f_{\max}$ are the width, the central, the minimum and the maximum frequencies of a function of a spectral concentration of a signal accordingly. Such signal transfers an amount of information about the object of research in $\mu / \mu_n \gg 1$ time greater, than well known conventional narrow-band and wide-band signals, for which their wideband index obeys to a condition $\mu_n \ll 1$. The payment for received advantages appears an inapplicability of conventional methods of generation, radiation, detection, reception and processing for UWB signals. The latter is connected with the fact that the function of a spectral concentration of UWB signal is distorted, so, and basic information parameter of a signal – its form [1].

2. Wavelet Analysis

One of possible paths of escaping of the given situation is the application of new mathematical research techniques of such signals. Expedient usage of the wavelet analysis is represented.

The wavelet analysis as a new mathematical method was designed in middle 80s of the twentieth century [2,3]. The essence of the method consists in decomposition of a parsed signal on basis of functions, restricted both in temporary, and in frequency area. This basis is remarkable because it is built with the help of stretching-compressions and shifts on the basis only one functions possessing special properties and called as wavelet. The wavelet-spectrum of a researched signal appears to be two dimensional: one axis – shift state, another – parameter of scaling. It allows to conduct the analysis with the necessary accuracy simultaneously both in a time-domain, and in frequency. The conventional two dimensional Fourier transform of such capability is not given.

3. Wavelets as UWB Signals

The wavelet analysis is convenient to apply to UWB signals, as it appears, that any wavelet is by definition a UWB signal. Therefore, the wavelet basis is an own basis for UWB signals [4].

Let's estimate wideband indexes for most frequently used wavelets. Such wavelets are HAAR-wavelet, WAVE-wavelet, FHAT-wavelet, DOG-wavelet and MHAT-wavelet [2, 3]. A kind of these wavelets in a time-domain, and also their spectra of the Fourier are adduced in Fig. 1.

Owing to principled perpetuity of a Fourier spectrum of UWB signal, as shown in [5], it is possible to determine the value $f_{\min}$ and $f_{\max}$ on the level

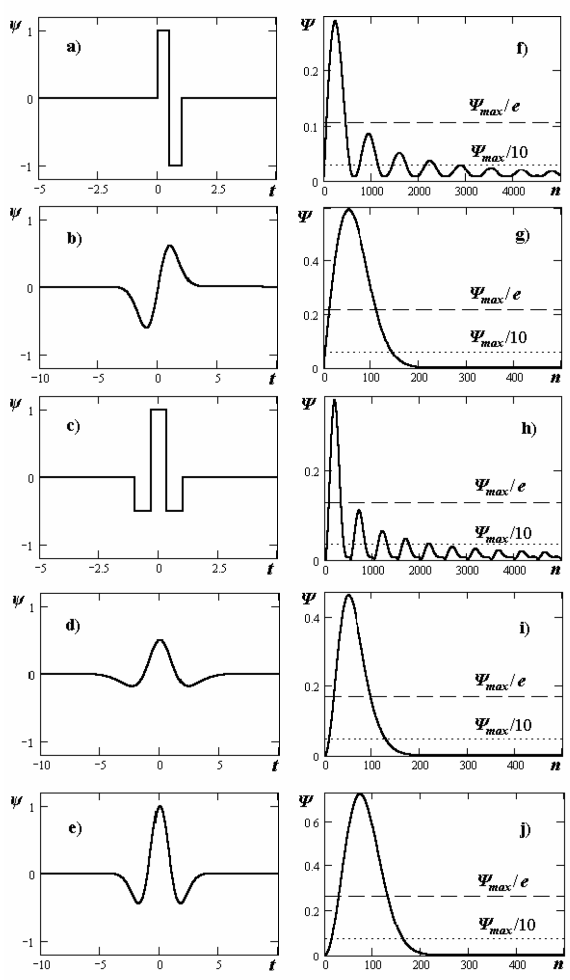

Fig. 1. *Wavelets and their Fourier spectra. In the time-domain: a) – HAAR-wavelet, b) – WAVE-wavelet, c) – FHAT-wavelet, d) – DOG-wavelet, e) – MHAT-wavelet.*
In spectral area: f) – HAAR-wavelet, g) – WAVE-wavelet, h) – FHAT-wavelet, i) – DOG-wavelet, j) – MHAT-wavelet

of decrease of a function of a spectral concentration of a signal in e or in 10 times concerning its maximum rating (see Fig. 1). The obtained value of a wideband index μ for mentioned above wavelets is given in Table 1. The value μ can also be determined by the approximated formula

$$\mu \approx \frac{4}{N}, \qquad (2)$$

where N is the quantity of half-waves of UWB signal [1]. The formula (2) works well only at $N > 2$, as at $N = 2$ $\mu \to 2$, that is characteristic for video signal, not of UWB signal. The values μ, calculated by the formula (2) are also adduced in Table 1.

Table 1. *Values of wideband index* μ *for miscellaneous wavelets*

№	Wavelet	μ		
		$\Psi_{\max}/e$	$\Psi_{\max}/10$	by the equation. (2)
1	HAAR	1,57	1,98	$\to 2$
2	WAVE	1,61	1,95	$\to 2$
3	FHAT	1,19	1,92	1,33
4	DOG	1,32	1,71	1,33
5	MHAT	1,28	1,68	1,33

From Table 1 it is clear, that at an estimation μ on a level of decrease of a function of a spectral concentration of wavelet in e time concerning its maximum level for HAAR-wavelet and for FHAT-wavelet of value appears rather low concerning remaining wavelets, though visually (see Fig. 1) their spectra decrease more slowly. If to use a level of decrease of a function of a spectral concentration not in e time, but in 10 times, the ratio between values μ for miscellaneous wavelets become more adequate. At the same time first of surveyed versions will better agree with results obtained on the formula (2).

4. Results of Calculations

The calculations have shown, that the wavelet spectra for different model of UWB signals (eight analytical models of UWB signals in a time-domain were used) are well enough localized on both by a variable. Different wavelets were applied: HAAR-wavelet, FHAT-wavelet, WAVE-wavelet, MHAT-wavelet, and DOG-wavelet. In Fig. 2 and Fig. 3 as examples the wavelet spectra of following models of UWB signals are presented:

$$s_1(t) = (1 - |2t - 1|)\sin(2\pi n t)\Theta(t), \qquad (3)$$

$$s_2(t) = (1 - t/\tau)\exp(-t/\tau)\eta(t), \qquad (4)$$

$$s_3(t) = (\exp(-\alpha_1 t) - \exp(-\alpha_2 t))\cos(\beta t)\eta(t). \qquad (5)$$

where

$$\Theta(t) = \eta(t) - \eta(t-1),$$

$\eta(t)$ is the Heavyside's function.

The function $s_1(t)$ can be used for simulation of UWB signals of a synthetic origin. The relation $s_2(t)$ describes an impulse of pressure [7], arising at air explosion, and $s_3(t)$ is known [8] as the model of whistlers. It is easy to show, that at $\beta = \sqrt{\alpha_1 \alpha_2}$ the expression (4) describes a UWB signal [6].

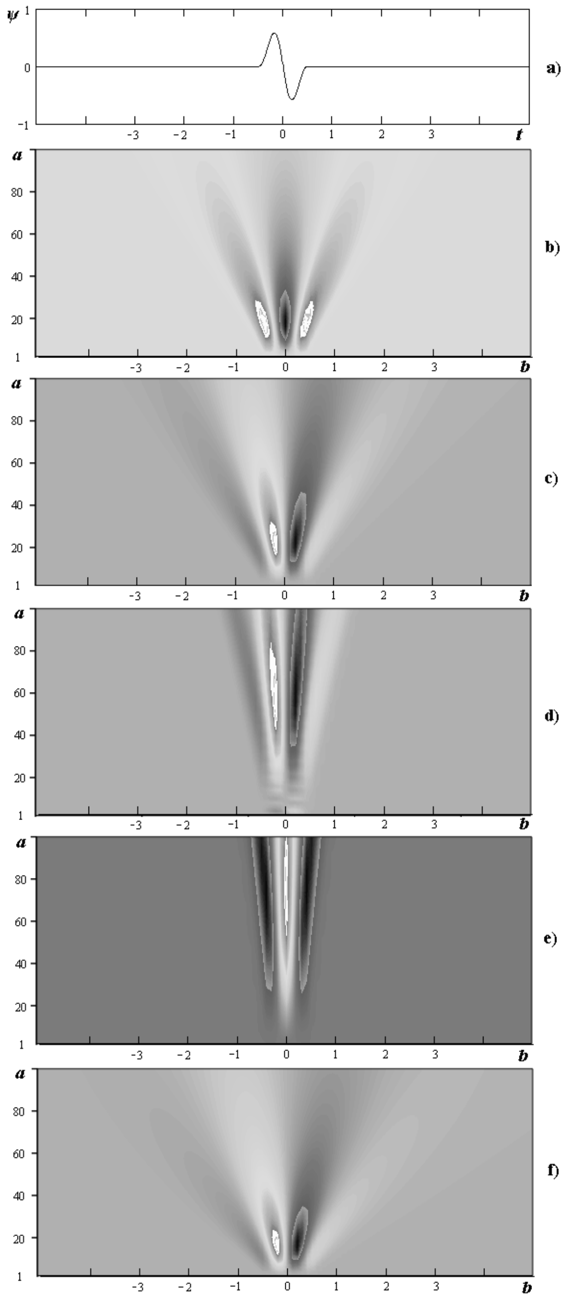

Fig. 2. *Wavelet spectra of UWB signal (a – UWB signal (model 1) in the time-domain), obtained for miscellaneous wavelets; b – WAVE-wavelet, c – MHAT-wavelet, d – FHAT-wavelet, e – HAAR-wavelet, f – DOG-wavelet.*
Here t is the time, a is the scaling parameter, b is the shift parameter

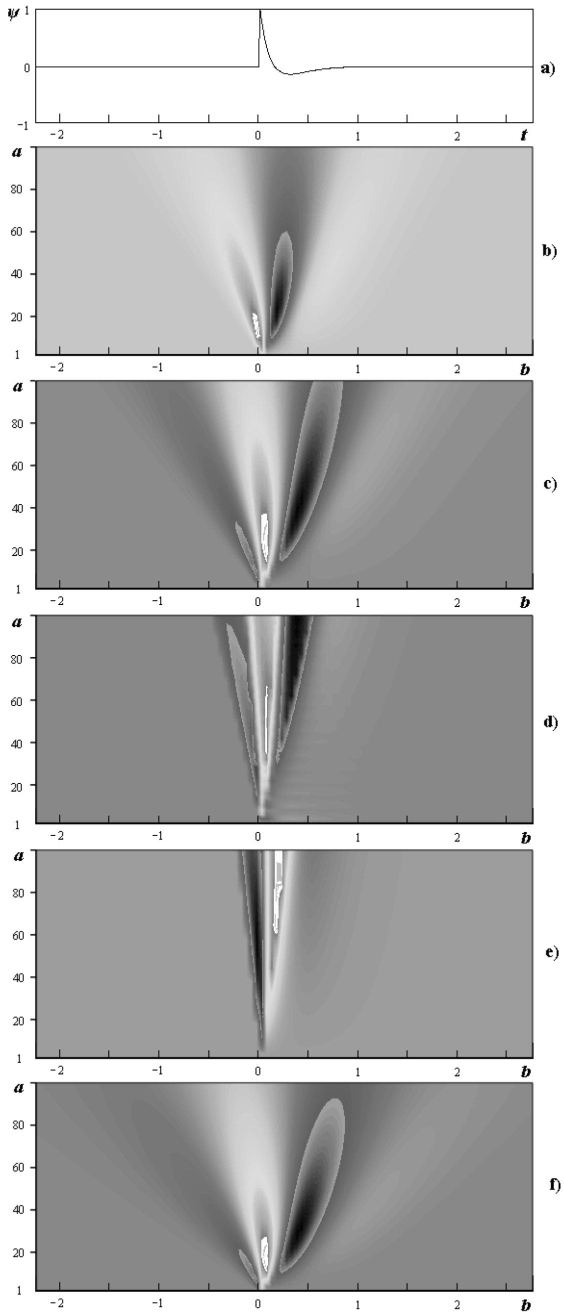

Fig. 3. *Wavelet spectra of UWB signal (a – UWB signal (model 2) in the time-domain), obtained for miscellaneous wavelets: b – WAVE-wavelet, c – MHAT-wavelet, d – FHAT-wavelet, e – HAAR-wavelet, f – DOG-wavelet.*
Here t is the time, a is the scaling parameter, b is the shift parameter

At the same time two dimensional Fourier spectrum of such signals is feebly localized along axis of frequencies (for this reason they and are named as ultra wideband signals). The latter is explained by the fact that finite UWB signals are much more similar to finite basic functions of wavelet transform, than to infinite basic functions of a Fourier transform.

Rather recent method of the wavelet analysis was successfully applied to the solution of a problem

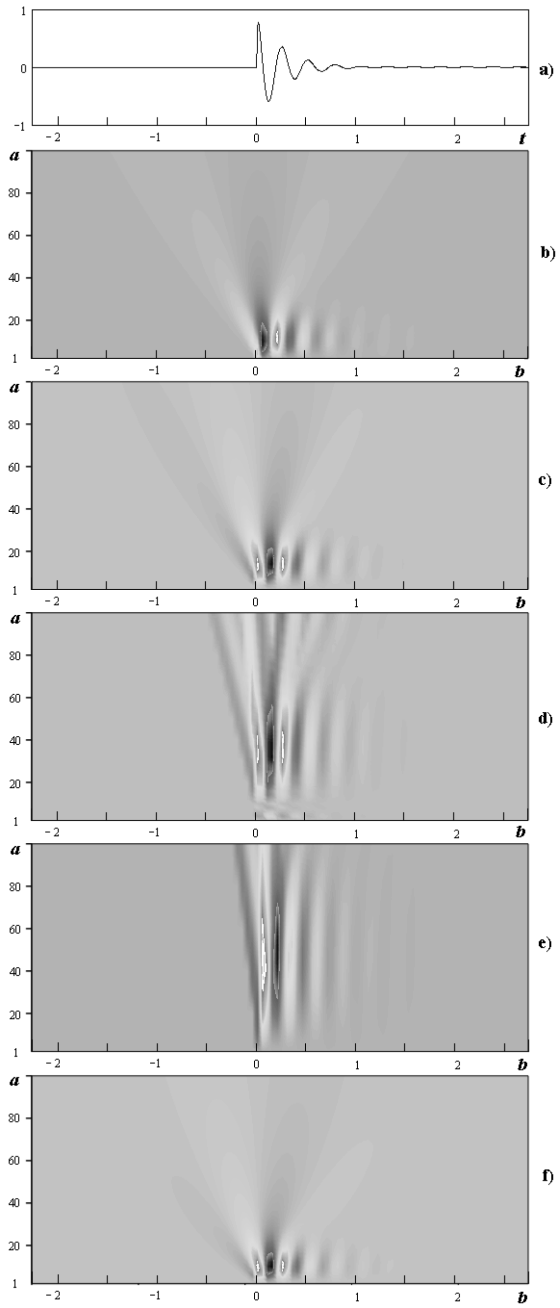

Fig. 4. *Wavelet spectra of UWB signal (a – UWB signal (model 3) in the time-domain), obtained for miscellaneous wavelets: b – WAVE-wavelet, c – MHAT-wavelet, d – FHAT-wavelet, e – HAAR-wavelet, f – DOG-wavelet.*
Here t is the time, a is the scaling parameter, b is the shift parameter

of detection of UWB signals on a background of the additive feebly correlated interference [6].

Thus, usage of the wavelet analysis as new mathematical method for research of UWB signals is expedient and perspective.

5. Conclusions

1. UWB signals are being more and more applied in a modern science and engineering and by virtue of the principled features usages require new mathematical methods for their analysis.
2. The wavelet analysis as a rather new mathematical method can successfully be applied to the description of UWB signals.

References

1. L.Y. Astanin, A.A. Kostylev. The Bases of the Ultra Wideband Radar Measurements.– M.: Radio and Communication, 1989.– 192 p. (in Russian).
2. The transforms and applications handbook / Editor-in-chief, A. Poularikas. USA, CRC Press, 1996, 1103 p.
3. M. Holschneider. Wavelets. An Analysis Tools. Oxford.– 1995.– 413 p.
4. O.V. Lazorenko. Proc. VIIth International Conference on Mathematical Methods in Electromagnetic Theory, June 2-5, 1998, Kharkov.– Kharkov.– 1998.– P. 621.
5. O.V. Lazorenko. Ph.D. thesis. Kharkov, KSU, 1999, 201 p. (in Russian).
6. O.V. Lazorenko, S.V. Lazorenko, L.F. Chernogor. Radio Physics and Radio Astronomy.– 2002.– **7**.– № 1.– pp. 46-63. (in Russian).
7. Detection of nuclear-weapon tests. Theme issue TIIER. 1965, V. **53**, №12. (in Russian).
8. J.V. Stennikov, V.V. Dobrjansky. Problems of diffraction and wave diffusion. Leningrad, I LIE,– Iss. 9, 1969, pp. 138-145. (in Russian).

ВЕЙВЛЕТНЫЙ АНАЛИЗ И СВЕРХШИРОКОПОЛОСНЫЕ СИГНАЛЫ

Л.Ф. Черногор, О.В. Лазоренко, С.В. Лазоренко

Вейвлет-анализ предлагается использовать для описания сверхширокополосных сигналов. Обосновывается целесообразность такого подхода. Показывается, что вейвлеты являются сверхширокополосными сигналами. Вычисляются показатели широкополосности наиболее часто используемых вейвлетов. Обсуждаются преимущества вейвлет-анализа перед традиционными методами при исследовании сверхширокополосных сигналов. В качестве примера рассматриваются вейвлет-спектры некоторых модельных сверхширокополосных сигналов.

ВЕЙВЛЕТНИЙ АНАЛІЗ ТА НАДШИРОКОСМУГОВІ СИГНАЛИ

Л.Ф. Черногор, О.В. Лазоренко, С.В. Лазоренко

Вейвлет-аналіз пропонується застосовувати для описання надширокосмугових сигналів. Обгрунтовано доцільність такого підходу. Показано, що вейвлети є надширокосмуговими сигналами. Обчислюються показники широкосмуговості вейвлетів, що використаються найчастіше. Обговорено переваги вейвлет-аналізу над традиційними методами при дослідженні надширокосмугових сигналів. Як приклад розглянуто вейвлет-спектри деяких модельних надширокосмугових сигналів.