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IMPULSE SOURCE IN THE VICINITY OF A CONVEX IMPEDANCE BODY: MINIMIZATION OF THE FIELD IN THE SHADOW REGION

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The work deals with the optimization problem of impulse source field minimization in the shadow region of a convex body with variable surface impedance. The optimal impedance is searched using uniform asymptotic methods and Ritz method for functional minimization. Numerical results are presented for the Gaussian impulse source.

1. Introduction

The modern means of transport, such as aircrafts, can contain several dozens transmitting and receiving antennas aboard. Each of these antennas is the potential source of the interference for the others. The most difficult situation arises when the transmitting and receiving signals are wideband, and especially if the signals' spectrums are overlapped.

That is why the developers have to take appropriate steps to decrease the undesirable reciprocal effects. This can be made, for example, by means of the optimal positional relationship, or directional patterns correction, or by means of the various coverings.

This work presents a method of the body impedance (or covering) distribution determination, under which the field of the first antenna is the lowest on the second one. It is based on the well-known asymptotic methods of field determination – Geometrical Theory of Diffraction (GTD), Uniform Asymptotic Theory (UAT) [1] and Ritz method for functional minimization [2].

2. Problem Setting

Let us consider the following problem: The point of observation M is situated in the shadow region relative to point source M_0 (Fig. 1).

We will study the case of *E*-polarization. The case of H-polarization can be considered in a similar manner, but with the significant restriction: the impedance of the body can't tend to infinity.

The body is bounded by the smooth curve l. The curvature radius of l is $\rho(s)$, where s is the natural parameter of l. The impedance of the body $g(s)$ can be any smooth complex function of s with the only restriction $|g(s)| = O(1)$. Enter the coordinates (s,n) where n is the length of the perpendicular dropped to the body from a point, and s is the natural parameter of the meet point of the body and the perpendicular.

The point source is supposed to emit the following wave:

$$U(\vec{r},t) = U_0 F(t)\delta(\vec{r} - \vec{r}_0), \qquad (1)$$

where U_0 is the amplitude of the source, $F(t)$ – arbitrary time-dependent multiplier, and $\delta(\vec{r} - \vec{r}_0)$ – the two-dimensional delta function.

We are looking for the function $g(s)$, which minimizes the field of the source at the observation point M.

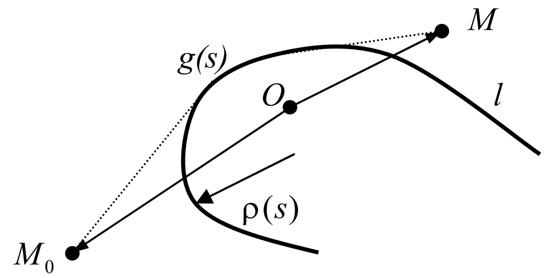

Fig. 1.

3. Solution

The pulse radiation of the source can be presented in time domain as Fourier integral:

$$U(\vec{r},t) = \frac{U_0\delta(\vec{r}-\vec{r}_0)}{2\pi}\int_{-\infty}^{\infty}\Phi(\omega)\exp(-i\omega t)\,d\omega\,, \quad (2)$$

where ω, $\Phi(\omega)$ are the cyclic frequency and the Fourier amplitude respectively. The field in the shadow region can be then presented in the form:

$$U(\vec{r},t) = \frac{U_0}{2\pi}\int_{-\infty}^{\infty}\Phi(\omega)\exp(i\omega t)\,G(\vec{r},\vec{r}_0 \mid \omega)\,d\omega\,, \quad (3)$$

where $G(\vec{r},\vec{r}_0 \mid \omega)$ is the Green function of the problem, identical to the initial, but with the monochromatic source. We assume that the Fourier amplitude $\Phi(\omega)$ tends to zero outside the region $|\omega-\omega_0| \sim N/\tau$, where τ is the characteristic time of the impulse, and therefore the interval of integration in (3) can be decreased to the interval that is not likely to exceed $\left[\omega_0-\omega_0/2, \omega_0+\omega_0/2\right]$. In this case the solution is valid if $a\omega/2c \sim 10$ or larger (a is the characteristic dimension of the body).

The Green function $G(\vec{r},\vec{r}_0 \mid \omega)$ can be calculated using GTD formulas for the creeping waves if both the point source and the observation point are far enough from the body. If the points are close to the body, the asymptotic formulas of V.M. Babich and V.S. Buldyrev [1] should be used:

$$U(M) = \sum_{n=-\infty}^{\infty}\sum_{p=0}^{\infty}\Gamma_p(r_0,\varphi_0;r,\varphi+2\pi n;k); \quad (4)$$

$$\Gamma_p(r_0,\varphi_0;r,\varphi;k) = \frac{1}{2i}\left(\frac{2}{k}\right)^{1/3}\left[\rho(s)\rho(s_0)\right]^{-1/6}\times$$
$$\left[w_1'(\xi_p)\right]^{-2}e^{ik(s-s_0)}\exp\left(i\xi_p\left(\frac{k}{2}\right)^{1/3}\int_{s_0}^{s}\frac{ds}{\rho^{2/3}(s)}\right)\times$$
$$\exp\left(\int_{s_0}^{s}\frac{ds}{\rho(s)g(s)}+\right.$$
$$\left.+\frac{i}{6k^{1/3}}\left[\frac{\rho'(s)}{\rho(s)}\nu^2-\frac{\rho'(s_0)}{\rho(s_0)}\nu_0^2\right]\right)\times$$
$$\exp\left(i\frac{\alpha_{10}(s)-\alpha_{10}(s_0)}{k^{1/3}}+O(k^{-2/3})\right)\times$$
$$w_1(T(\xi_p,M_0))\,w_1(T(\xi_p,M)),$$

where $w_1(x)$, $w_1'(x)$ are the Airy function of first order and its derivative $w_1(z) = 2e^{i\frac{\pi}{6}}\frac{\sqrt{\pi}}{2}Ai(ze^{2\pi i/3})$; ξ_p is the p-th root of the equation $w_1(\xi_p)=0$; $\nu = nk^{2/3}$, and α_{10}, $T(\xi,M)$ are defined by the following formulas:

$$\alpha_{10}(s) = 2^{1/3}\xi_p^{\,2}\times$$
$$\int_0^s \rho^{-4/3}(s)\left[\frac{1}{60}+\frac{4}{135}\rho'^2(s)-\frac{2}{45}\rho(s)\rho''(s)\right]ds$$

$$T(\xi,M) = \xi-\nu\left(\frac{2}{\rho(s)}\right)^{1/3}-$$
$$\frac{i}{(k/2)^{1/3}\rho^{1/3}(s)g(s)}+O(k^{-2/3}).$$

The represented formula is inapplicable when $|g(s)|\to 0$, and therefore the situations with small impedances should be avoided during the optimization procedures. It is also inapplicable for the low frequencies ω and, consequently, for the low frequencies ω_0.

In order to solve the initial problem we have to minimize one of the following functionals:

$$F_1(g(s)) = |U(g(s);M,M_0)|,$$
$$F_2(g(s)) = \max_i|U(g(s);M+\delta M_i,M_0)|, \quad (5)$$
$$F_3(g(s)) = \max_i\left|U\left(g(s);M,M_0+\delta M_0^{\,i}\right)\right|.$$

The functionals $F_{2,3}$ can be used if the source or the observation point varies within the defined limits. They also allow estimating the stability of the results against the deviations of the initial conditions.

The problem of the functional minimization can be reduced to the problem of function minimization if we assume that the impedance can be expanded in the following series:

$$g(s) = \sum_p^N a_p f_p(s) + R_N(s), \quad (6)$$

where $f_p(s)$ are members of the set of orthogonal functions that is complete in the space of the functions looked for, and $R_N(s)$ is the remainder [2]. When we put (6) in (4), we don't further need to find the function $g(s)$, but a finite number of the coefficients a_p which minimize one of the functions (5). There is a great variety of function minimization methods. In the present work the method of Nelder and Mead [3] has been used, though the other methods, for example [4], are also applicable.

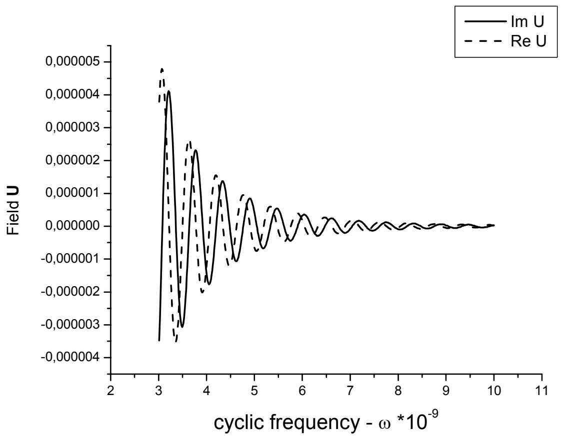

Fig. 2.

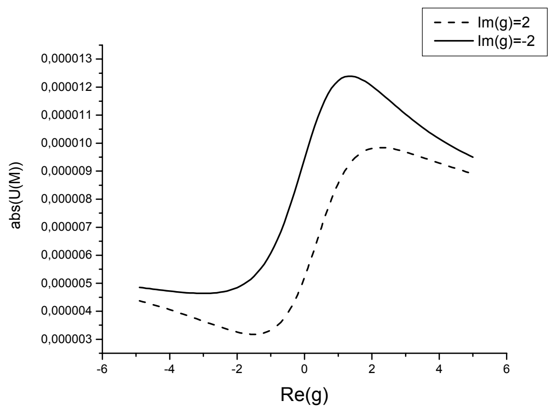

Fig. 4.

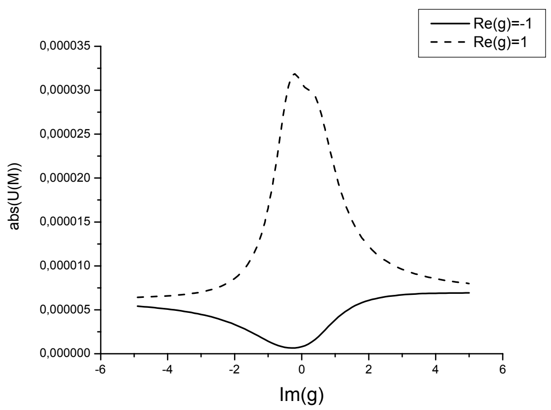

Fig. 3.

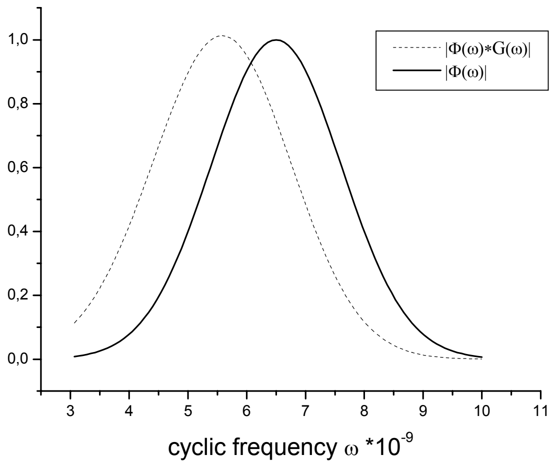

Fig. 5.

4. Numerical Results

In order to test the method, it was applied to the problems of scattering on circular, elliptic and parabolic cylinders. Fig. 2 shows the typical behavior of the Green function (4) while the frequency varies and the other parameters are constant. ($\rho = const$; $k\rho = 40..120$; $s_0 = 0$; $s = \pi\rho$; $n = 0.1\rho$; $n_0 = 0.2\rho$).

The field damps exponentially with the frequency increasing and in the limit tends to zero, which corresponds to the geometrical optics. The amplitude of the field depends upon the impedance of the body and the geometry of the problem.

The amplitude dependencies on the real and imaginary parts of the impedance are shown in Fig. 3,4 ($s_0 = 0$; $s = \pi$; $n = n_0 = 0.1$; $\rho = 1$; $k\rho = 10$, monochromatic source). Fig. 3 corresponds to the case $\mathrm{Re}(g(s)) = \mathrm{const} = \pm 1$, and the Fig. 4 – to the case $\mathrm{Im}(g(s)) = \mathrm{const} = \pm 2$. The dependencies have extremums at the small real (imaginary) parts of the impedance, and have the common limit if $\mathrm{Re}(g) \to \pm\infty$ ($\mathrm{Im}(g) \to \pm\infty$). This allows to solve both the problem for minimization and maximization of the field. It is evident from Fig. 3, 4 that the minimum of the field for the circle can be reached only if the real part of the impedance becomes negative, at least in a small sector, and the maximum – only if the real part becomes positive, at least in a small sector.

Consider then the simplest impulse source with the following time-dependent multiplier:

$$F(t) = \cos(\omega_0 t)\exp\left(-(t/\tau)^2\right).$$

The Fourier amplitude of the signal is:

$$\Phi(\omega) = \tau\sqrt{\pi}\exp\left(\frac{-(\omega-\omega_0)^2\,\tau^2}{4}\right).$$

At the observation point the spectrum of the signal shifts toward the lower frequencies as a consequence of the damping of the Green function (4). This situation is shown in Fig. 5 (the geometry of the system is the same as in Fig. 2).

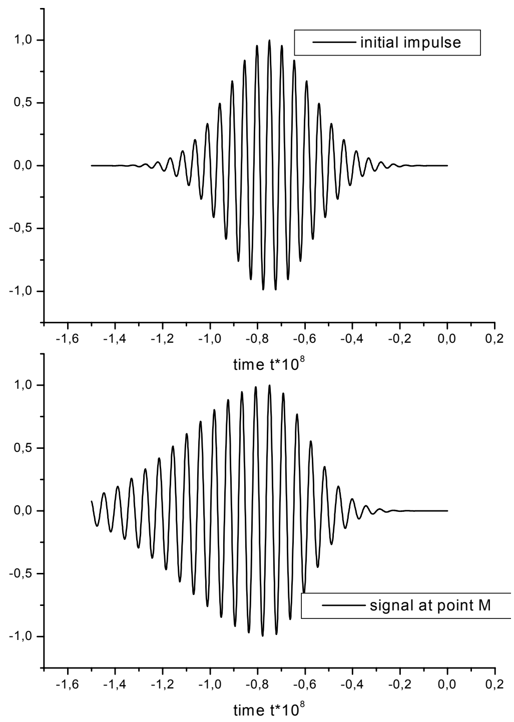

Fig. 6.

As a result of the spectrum shift and distortion, the signal at the observation point has lower carrier frequency, than the initial signal, and the pulse duration increases. The waveforms of the initial signal and the signal at the point M are compared in Fig. 6 (time axes are shifted).

The results of the minimization procedures for the case with the single creeping wave are close to each other and to the result of monochromatic source irrespective to the method of time averaging (the maximum of the amplitude during the pulse time or root-mean-square sum of the amplitudes). Fig. 7 shows the impedance distribution that minimizes the field in the vicinity of the parabolic cylinder ($kF = 10...20$, where F is the focal distance; $s_0 = -F$; $s = F$; $n_0 = 0.05F$; $n = 0.1F$; $\mathrm{Re}(g(s)) = -1$).

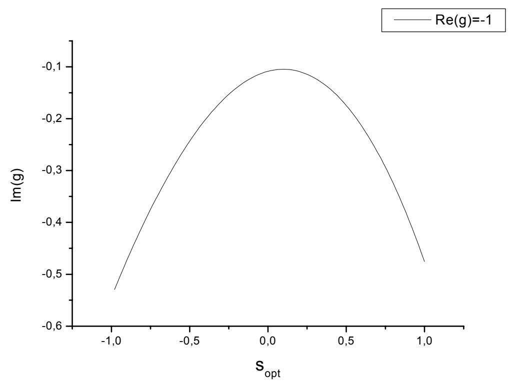

Fig. 7.

For the cases with several creeping waves the results of the optimization procedures differ sharply. In these cases with the monochromatic source, the minimization can be reached using the interference of the creeping waves. For the impulse sources, it can take place only if the impedance is time-dependent.

5. Conclusions

The proposed method of field minimization in the shadow region has shown that it can be used for various optimization procedures, provided that $ka \sim 20$ or larger, where a is the typical dimension of the scatterer. It can also be generalized to the 3-dimensional case and improved by allowing for surface waves in the case $\mathrm{Im}\, g(s) < 0$; $\mathrm{Re}\, g(s) > 0$. As the formula (4) actually presents the Green function, the method can be easily modified to deal with distant sources.

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ИМПУЛЬСНЫЙ ИСТОЧНИК ВБЛИЗИ ГЛАДКОГО ИМПЕДАНСНОГО ТЕЛА: МИНИМИЗАЦИЯ ПОЛЯ В ЗОНЕ ТЕНИ

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В настоящей работе представлен метод определения оптимального покрытия поверхностей (поверхностного импеданса), при котором достигается минимум поля точечного импульсного источника в точке наблюдения, расположенной в тени относительно этого источника. Метод основан на геометрической теории дифракции и равномерных асимптотических методах, а также на методе минимизации функционалов Ритца.

ІМПУЛЬСНЕ ДЖЕРЕЛО БІЛЯ ГЛАДКОГО ІМПЕДАНСНОГО ТІЛА: МІНІМІЗАЦІЯ ПОЛЯ У ЗОНІ ТІНІ

А.О. Звягінцев, О.І. Іванов, Д.В. Катков

У роботі представлено метод визначення оптимального покриття поверхонь (поверхневого імпедансу), при якому досягається мінімум поля імпульсного джерела у точці спостереження, яка розташована у тіні відносно цього джерела. Метод засновано на геометричній теорії дифракції та рівномірних асимптотичних методах, а також на методі мінімізації функціоналів Рітца.