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THE MODELS OF THE "VOIDS" IN THE UNIVERSE

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The astronomical observations of the last years show that there are regions in the Universe with much lower density of matter, than in the surrounding space. Theoretical studies of the regions (voids) in the model of the expanding Universe are carried out in different directions. In this paper the voids have been built by means of matching the Tolman and Friedman solutions. The The Lichnerovich-Darmour matching conditions are used. It is shown that in the expanding Universe with flat space the voids cannot exist. So, we have the Friedman Universe with voids, which are described by the Tolman solution. The models of the voids in the Friedman Universe with a negative spatial curvature are built.

1. Introduction

The astronomical observations of last years show that there are regions in the Universe with much lower density of matter than than in the surrounding space. Theoretical studies of these regions (voids) in the models of the expanding Universe are carried in different directions: small perturbations of homogeneous Universe; the use of the Einstein-Straus model; the use of the Tolman solution for a nonhomogeneous dust; consideration of the boundary of the void as a thin wall ([1, 2, 3]). In this paper we use the Tolman spherically symmetric dust solution for the description of the voids space-time, and Friedman solution for the description of the space-time of the surrounding Universe.

2. Tolman solution

The Tolman solution is following:

$$ds^2 = dt^2 - \frac{r'^2(R,t)}{f^2(R)}dR^2$$
$$-r^2(R,t)(d\Theta^2 + \sin^2\Theta d\varphi^2), \qquad (2.1)$$

where

$$r(R,t) = \frac{m(R)}{1-f^2(R)} \times \begin{cases} \sin^2(\alpha/2), & \text{for } f^2(R) < 1, \\ -\sinh^2(\alpha/2) & \text{for } f^2(R) > 1, \end{cases} \qquad (2.2)$$

$$t - t_0(R) = \frac{m(R)}{2|1-f^2(R)|^{3/2}} \times \begin{cases} \alpha - \sin\alpha, & \text{for } f^2(R) < 1, \\ \sinh\alpha - \alpha, & \text{for } f^2(R) > 1, \end{cases} \qquad (2.3)$$

$$r(R,t) = \left[\pm\frac{3}{2}m(R)^{1/2}(t - t_0(R))\right]^{2/3}. \qquad (2.4)$$

for $f^2(R) = 1$. The velocity of light $c = 1$. The prime denotes ∂/∂_R. $m(R)$, $f(R)$ and $t_0(R)$ are the arbitrary functions of the integration, $m(R)$ is the mass function – the active gravitational mass of the dust ball with radial coordinate R, the function $f(R)$ defines geometry of a three-dimensional part of Tolman solution, $t_0(R)$ determines the time of the collapse.

The energy density:

$$\varepsilon(R,t) = \frac{1}{8\pi\gamma}\frac{m'(R)}{r^2(R,t)r'(R,t)}. \qquad (2.5)$$

The particular case of the Tolman solution is the Friedman one for which $t_0(R) = 0$ for all types of the space curvature at $f^2(R) = \cosh^2(R), m(R) = m_F(R) = a_0\sinh^3(R)$ for solutions of a hyperbolic type, at $f^2(R) = \cos^2(R), m(R) = m_F(R) = a_0\sin^3(R)$ for solutions of an elliptic type and at $f^2(R) = 1, m(R) = m_F(R) = b_0R^3$ for solutions of a parabolic type.

As a matching surface, boundary of "void", we shall select a hypersurface $R = R_b = const$. From the Lichnerovich-Darmour matching conditions follows, that on such a hypersurface next conditions are fulfilled:

$$r_T(R_b, t_T) = r_F(R_b, t_F), \qquad (2.6)$$

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$$f_T(R_b) = f_F(R_b), \qquad (2.7)$$

$$m_T(R_b) = m_F(R_b), \qquad (2.8)$$

where the symbols T and F designate the magnitudes relared to the Tolman and Friedman space - time, accordingly.

From conditions (2.6) – (2.8) follows, that the space curvature of the "void" should have the same sign, as space curvature of the external space. Without loss of generality it is possible to consider the same function $f(R)$ in both metrics. From matching conditions (2.6), (2.7) and (2.8) we obtain, that on the matching surface $R = R_b$, $\alpha_T(R_b, t_T) = \alpha_F(t_F)$, therefore, $t_T - t_0(R_b) = t_F$. Thus we can make the conclusion, that at $t_0(R) \neq 0$ the substance in the "voids" is "older", than in the surrounding space ([4, 5]).

3. The "Voids" in the Friedman world with zero space curvature

The average density of matter in the "void"is given by:

$$\overline{\varepsilon}_T = \overline{\varepsilon}(t_T) = \frac{M_T}{V_T}, \qquad (3.1)$$

where the mass

$$M = \int_0^{R_b} \varepsilon\, \sqrt{-g}\, dR\, d\Theta\, d\varphi, \qquad (3.2)$$

the volume of the "void"

$$V = \int_0^{R_b} \sqrt{-g}\, dR\, d\Theta\, d\varphi. \qquad (3.3)$$

For the Tolman solution, with (2.1) – (2.5) and (3.2), (3.3), the expressions for a mass and volume can be written as follows:

$$M = \int_0^{R_b} \frac{m'_T(R)}{f_T(R)}\, dR;$$

$$V = \int_0^{R_b} \frac{r_T^2(R,t) r'_T(R,t)}{f_T(R)}\, dR. \qquad (3.4)$$

By substitution of eq. (3.4) in (3.1) we obtain

$$\overline{\varepsilon}_T = \frac{3 m_T(R_b)}{r_T^3(R_b, t_T)}. \qquad (3.5)$$

The eq. (3.5) for average density is valid for any Tolman solution, including the Friedman solution. For the homogeneous Friedman space

$$\overline{\varepsilon}_F = \varepsilon_F(t) = \frac{m'_F(R)}{r_F^2 r'_F} = \frac{3 m_F(R)}{r_F^3(R, t_F)}. \qquad (3.6)$$

Since $\varepsilon_F(t)$ does not depend on R, we can substitute $R \to R_b$

$$\overline{\varepsilon}_F = \varepsilon_F(t) = \frac{3 m_F(R_b)}{r_F^3(R_b, t_F)}. \qquad (3.7)$$

We see that in the parabolic Friedman model the voids cannot exist, because the homogeneous energy density in the external space and the average density in the internal space are the same.

4. Model of "voids" in the Friedman Universe with negative space curvature

For the description of the external space we choose the Friedman solution:

$$r_F(R, t_F) = a_0 \sinh(R) \sinh^2(\alpha_F/2), \qquad (4.1)$$

$$t_F = \frac{a_0}{2}(\sinh \alpha_F - \alpha_F). \qquad (4.2)$$

Let us choose the mass function for the Tolman solution as

$$m_T(R) = a_0 \times \left\{ \frac{sinh^{n+1} R}{sinh^{n-2} R_b} \frac{\Psi(R)}{\Psi(R_b)} + [F(R) - F(R_b)]^L \right\}, \qquad (4.3)$$

where $\Psi(R)$, $\Psi(R_b)$ and $F(R)$ are arbitrary functions which are not having of singularities at $0 \leq R \leq R_b$; n, L — arbitrary numbers.

Let us consider the case:

$$m_T(R) = a_0 \frac{\sinh^{n+1} R}{\sinh^{n-2} R_b}, \qquad (4.4)$$

$$\frac{\Psi(R)}{\Psi(R_b)} = 1, \quad F(R) = F(R_b) = 0.$$

Then

$$r_T(R, t_T) = a_0 \frac{\sinh^{n-1} R}{\sinh^{n-2} R_b} \sinh^2(\alpha_T/2), \qquad (4.5)$$

$$t_T - t_0(R) = \frac{a_0}{2} \left[\frac{\sinh R}{\sinh R_b} \right]^{n-2} (\sinh \alpha_T - \alpha_T). \qquad (4.6)$$

Thus for the chosen mass function (4.4) there are two different kinds of solutions: for $n > 2$ and $n < 2$ (case $n = 2$ corresponds the Friedman solution (4.1), (4.2)).

4.1. *Sizes of the "voids"*

The astronomical data of observable "voids" in the Universe testify that their sizes change in the range from $25Mpc$ up to $100Mpc$ (the greatest observable "void" has a size $124Mpc$). The size of the "void" depends of the value of R_b

$$r_T(R_b,t_T)=r_F(R_b,t_F) \\ =a_0\sinh R_b\sinh^2(\alpha_F/2), \quad (4.7)$$

where α_F is given by (4.1), (4.2), $a_0 = 3,25\cdot 10^3 Mpc$. From this fact we can arrive at the conclusion, that the models of the "voids", satisfying observational data, should be characterized by the value of R_b, which changes in range: $0.004 < R_b \le 0.02$ (the greatest "void" has $R_b = 0.023$).

4.2. *Models of the "voids" for the case $n = 1$*

For our model:

$$m_T(R) = a_0 \sinh^2 R \sinh R_b, \quad (4.8)$$

$$r_T(R,t_T) = a_0 \sinh R_b \sinh^2(\alpha_T/2), \quad (4.9)$$

$$t_T - t_0(R) = \frac{a_0}{2}\frac{\sinh R_b}{\sinh R}(\sinh\alpha_T - \alpha_T). \quad (4.10)$$

In the given class of this unique satisfactory solution (for whole n), as in all solutions with $n < 1$, there exists the singularities: at $R \to 0$ $r \to \infty$. Then the mass of a "void" is

$$M_T(R_b)=\int_0^{R_b}\frac{m'_T(R)}{f_T(R)}dR \\ =2a_0\sinh R_b(\cosh R_b - 1). \quad (4.11)$$

The volume of the configuration is:

$$V_T(R_b,t_T) = \int_0^{R_b}\frac{r_T^2(R,t_T)r'_T(R,t_T)}{f_T(R)}dR$$

$$= \left.\frac{r_T^3(R,t_T)}{3f_T(R)}\right|_0^{R_b} + \int_0^{R_b}\frac{r_T^3(R,t_T)}{3}\frac{f'_T(R)}{f_T^2(R)}dR$$

$$= a_0^3\left[A_1(R_b,t_T) + B_1(R_b,t_T)\right]. \quad (4.12)$$

The function $A_1(R_b,t_T)$ is calculated at the boundary of the the "void", therefore the function $\widetilde{\alpha}_T$ at the boundary

$$t_T - t_0(R_b) = \frac{a_0}{2}(\sinh\widetilde{\alpha}_T - \widetilde{\alpha}_T) = t_F. \quad (4.13)$$

Therefore, we can substitute $\widetilde{\alpha}_T \to \alpha_F$, and function $A_1(R_b,t_T) \equiv A_1(R_b,t_F)$. For calculation α_T in the function $B(R_b,t_T)$ it is necessary to use equation (4.10).

The Friedman solution gives the following value of the mass and the volume of the configuration:

$$M_F(R_b)=\int_0^{R_b}\frac{m'_F(R)}{f_F(R)}dR \\ =\frac{3a_0}{2}\left(\frac{\sinh 2R_b}{2} - R_b\right),$$

$$V_F(R_b,t_F) = \int_0^{R_b}\frac{r_F^2(R,t_F)r'_F(R,t_F)}{f_F(R)}dR$$

$$= \left.\frac{r_F^3(R,t_F)}{3f_F(R)}\right|_0^{R_b} + \int_0^{R_b}\frac{r_F^3(R,t_F)}{3}\frac{f'_F(R)}{f_F^2(R)}dR$$

$$= a_0^3\left[C_1(R_b,t_F) + D_1(R_b,t_F)\right], \quad (4.14)$$

where for the calculation of $\alpha_F(t_F)$ in the functions $C_1(R_b,t_F)$ and $D_1(R_b,t_F)$ we used the condition

$$t_F = \frac{a_0}{2}(\sinh\alpha_F - \alpha_F). \quad (4.15)$$

Therefore, $A_1(R_b,t_F) = C_1(R_b,t_F)$. The constructed model can be considered as a model of the "void" if the following condition are fulfilled

$$\frac{\overline{\varepsilon}_T}{\overline{\varepsilon}_F} = \frac{M_T(R_b)}{M_F(R_b)}\frac{V_F(R_B,t_F)}{V_T(R_b,t_T)} \ll 1. \quad (4.16)$$

It is necessary to note that at $t_0(R) = const$ the "voids" by definition (4.16) do not exist, since the value $\overline{\varepsilon}_T/\overline{\varepsilon}_F = 1$. The "Voids" can exist only at $t_0(R) \ne const$.

Let us consider some models with $t_0(R) = R$ (table 1 and table 2). In the tables the numerical calculations of the evolution of basic parameters of the considered configurations are given. The time t_F is dimensionalless time of the observer in the Friedman Universe devided by the constant quantity $3\cdot 10^{17}c$ ($t_F = 1$ - meets the present time).

R_b	t_F	V_T/V_F	M_T/M_F	$\overline{\varepsilon}_T/\varepsilon_F$
	0.01	1.001		0.999
	0.1	1.013		0.987
	0.25	1.2		0.83
0.005	0.5	3.33	1.0	0.3
	0.75	12.5		0.08
	1	41.7		0.024
	1.5	303.03		0.0033

Table 1.

R_b	t_F	V_T/V_F	M_T/M_F	$\bar{\varepsilon}_T/\varepsilon_F$
0.01	0.01	1.001	1.0	0.999
	0.1	1.011		0.989
	0.25	1.13		0.882
	0.5	2.36		0.423
	0.75	7.35		0.136
	1	22.22		0.045
	1.5	156.25		0.0064

Table 2.

Consider the evolution of the "voids" since time $t_F = 10^{-2} \times 3 \cdot 10^{17}c$. In all surveyed models the "voids" arise not earlier than in the time equal to $0.75 \cdot 3 \cdot 10^{17}c$. On the basis of it we can state that the "voids" generated in the near past and prolong to exist in the future. To constructed models of the "voids" with mass function (4.8) with grater overfull of the density it is possible to consider $t_0(R)$ as the function of a higher order (for example, $t_0(R) = R^3$). But it is possible only for the greatest "voids" $R_b \geq 1$. The volumes of the configurations described by the Tolman solution are considerably greater than the volumes described by Friedman ones (at same R_b). So we obtain models of the "voids" satisfying to the necessary requirements.

4.3. Models of the "voids" for the case $n > 2$.

Let us consider model of the "voids" for a case $n = 3$:

$$m_T(R) = a_0 \frac{\sinh^4 R}{\sinh R_b}, \qquad (4.17)$$

$$r_T(R, t_T) = a_0 \frac{\sinh^2 R}{\sinh R_b} \sinh^2(\alpha_T/2), \qquad (4.18)$$

$$t_T - t_0(R) = \frac{a_0}{2} \frac{\sinh R}{\sinh R_b} (\sinh \alpha_T - \alpha_T). \qquad (4.19)$$

Then the mass of "voids":

$$M_T(R_b) = \int_0^{R_b} \frac{m_T'(R)}{f_T(R)} dR = a_0 \frac{8}{3} \sinh^2(R_b/2) \times \tanh(R_b/2)(\cosh R_b + 2). \qquad (4.20)$$

The volume of the configuration described by Tolman solution is:

$$V_T(R_b, t_T) = \int_0^{R_b} \frac{r_T^2(R, t_T) r_T'(R, t_T)}{f_T(R)} dR = \left. \frac{r_T^3(R, t_T)}{3 f_T(R)} \right|_0^{R_b} + \int_0^{R_b} \frac{r_T^3(R, t_T)}{3} \frac{f_T'(R)}{f_T^2(R)} dR = a_0^3 (A_1(R_b, t_T) + B_2(R_b, t_T)), \qquad (4.21)$$

where the function $A_1(R_b, t_T)$ is calculated on the boundary of the "void", and for the calculation α_T in the function $B_2(R_b, t_T)$ it is necessary to use condition (4.19). For the calculation of the mass of the configuration and volume described by Friedman solution, we use expressions (4.14) – (4.15).

The evolution of the basic parameters of the "voids" models at time of the Friedman observer is given in the following tables (The investigation of models of the given class were is carried out for $5 \cdot 10^{-3} < R_b \leq 2 \cdot 10^{-2}$, $t_0(R) = R$):

These models describe the "voids" which existed only in the early Universe and to the present time all of them were filled by substance. By $t_0(R) = R^3$ for all the considered values of radiuses R_b a lifetime of "voids" is increased, but at the present time they not exist.

R_b	t_F	V_T/V_F	M_T/M_F	$\bar{\varepsilon}_T/\varepsilon_F$
0.007	0.000001	7.056	1	0.1417
	0.00001	1.62		0.9413
	0.001	1.0		0.9999
	0.1	1.0		1.0
	0.5	1.0		1.0
	1	1.0		1.0
	1.5	1.0		1.0

Table 3.

R_b	t_F	V_T/V_F	M_T/M_F	$\bar{\varepsilon}_T/\varepsilon_F$
0.02	0.000001	412.5	0.99	0.0024
	0.00001	5.13		0.193
	0.001	1.0		0.9999
	0.1	1.0		1.0
	0.5	1.0		1.0
	1	1.0		1.0
	1.5	1.0		1.0

Table 4.

5. Conclusion

The models of the "voids" are constructed. The Lichnerovich-Darmour matching conditions are fulfilled. The model of the "void" is described by Tolman spacetime, with the mass function (4.8), the external spacetime is the Friedman world. It follows from the matching conditions that the sign of the space curvature in the "void" should be same as in the Friedman one. But the size of the curvature can essentially differ.

It is shown, that in the Friedman world of a zero space curvature the "voids" cannot exist. The models of "voids" in the Friedman world of a negative space curvature are constructed. The matter inside the "void" always "older", than in the surrounding Universe. If the "voids" described by the Tolman solution with the mass function (4.8), they exist no more than one quarter from all lifetime of the Universe.

It is interesting to note, that the "voids" will be constructed because the volumes described by Tolman and Friedman solution are essentially various . The "voids" described by Tolman solution with the mass function (4.17) to the present time have do not exist.

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МОДЕЛИ "ПУСТОТ" ВО ВСЕЛЕННОЙ

М. П. Коркина, А. Н. Туринов

Астрономические наблюдения последних лет свидетельствуют о том, что во Вселенной существуют области, в которых плотность материи намного ниже, чем в окружающем пространстве. Данные области получили названия "пустот". Теоретическое исследование этих областей в моделях расширяющейся Вселенной осуществляется по нескольким направлениям. В этой работе "пустоты" строятся путем сшивки решений Толмена и Фридмана. В качестве условий сшивки используются условия Лихнеровича-Дармуа. Показано, что в плоской Вселенной "пустоты" отсутствуют. Построены модели "пустот" во фридмановской Вселенной отрицательной пространственной кривизны, описываемых решением Толмена.

МОДЕЛІ "ПУСТОТ" У ВСЕСВІТІ

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Астрономічні спостереження останніх років свідчать про те, що у Всесвіті існують області, в яких густина матерії є набагато нижчою, ніж у оточуючому просторі. Ці області одержали назву "пустот". Теоретичне дослідження цих областей у моделях Всесвіту, що розширюється, здійснюється у кількох напрямках. У даній роботі "пустоти" будуються шляхом зшивки рішень Толмена та Фрідмана. Як умови зшивки використовуються умови Ліхнеровича-Дармуа. Показано, що для плоского Всесвіту "пустоти" відсутні. Побудовані моделі "пустот" у Всесвіті Фрідмана від'ємної просторової кривизни, що описуються рішенням Толмена.