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Plasma Frequency Depression Coefficients for an Electron Beam Scattering on Metallic Surfaces

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The space charge depression coefficients technique is generalized for the case when a ribbon-like electron beam with an arbitrary distribution of the beam current is propagating at an angle with respect to a metallic surface. Analytical expressions for the depression coefficients are obtained and analyzed. Both local and non-local effect of space charge and their comparative contribution to the quasi-static field are considered.

1. Introduction

Space charge depression coefficients are widely used to account for influence of the geometry of the beam and the wave supporting system on the space charge field in electron devices [1-4]. Introduction of such coefficients allows one to describe conveniently the screening effect by conducting surfaces on the space charge field and to simplify the beam particle motion equation and its solution. In spite of a high capacity of modern computers, the computing of space charge field still remains a time-consuming procedure, and the problem of deriving the simplified expressions for the space charge field is still on the agenda. Besides, the depression coefficients technique gives one a simple approach to estimate analytically the extent of space charge influence on electron-wave interaction in a particular tube.

However, up to now a technique has been developed for computing and application of such coefficients only in the case of an electron beam parallel to the principal plane of the electrodynamical system (for example, a ribbon-like beam skimming over a grating or a cylindrical beam in a cylindrical waveguide) [1-4]. The aim of this work is to generalize the space charge depression coefficients technique for the case when an electron beam is propagating at an angle to a metallic surface. When solving this problem, we did not specify the beam modulation mechanism; it can be arbitrary to some extent. The final expressions for the space charge field can be used when solving a self-consistent problem of the beam interacting with the field, provided the modulation mechanism and its parameters are specified. The results obtained can be used for space charge effects description in such electron devices as the clinotron, M-type TWT, electron collectors, and others.

The paper is organized as follows: Section II gives the formulation of the problem; in Section III we derive an expression for space charge field and introduce the depression coefficients for an inclined beam; Section IV contains conclusions, and in Appendix we derive an easy-to-handle expression for the Green's function.

2. Problem Formulation

The geometry of the problem is shown in Fig. 1. An inclined electron beam with the thickness a is propagating with the velocity v_0 between two metallic planes and is scattered by one of them. The tilt angle is α, and D is the distance between the planes. Fig.1 presents the coordinate systems used: the axis $\tilde{z}, \tilde{y}$ are Cartesian coordinate axis referring to the geometry of the planes,

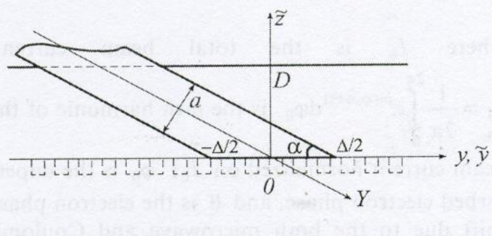

Fig. 1. *Schematics of the problem geometry with the coordinate systems used. The following configuration parameters are used: a is the beam thickness; α is the tilt angle; $\Delta = a/\sin\alpha$ is the area the beam covers on the metallic surface; D is the distance between the metallic planes*

whereas y and Y are coordinates as correspond to the beam geometry. We introduce the latter as follows:

$$y = \tilde{y} + \tilde{z}\cot\alpha, \qquad \alpha \neq 0,\ \pi$$

$$Y = -\tilde{z}/\sin\alpha.$$

Thus, we consider the two-dimensional formulation of the problem. Let us note that the lower plane can be a slow-wave supporting system, that is a grating. However, from the point of view of space charge field calculation the grating can be replaced by a plane surface when its period is less then the period of the synchronous wave [1]. We assume the beam modulated, the current and the charge densities, denoted as $j(\vec{r},t)$ and $\rho(\vec{r},t)$, respectively, being periodic functions of time:

$$j(\vec{r},t) = \sum_{n=1}^{\infty} j_n(\vec{r})\, e^{-in\omega t + in\beta_e Y},$$

$$\rho(\vec{r},t) = \sum_{n=1}^{\infty} \rho_n(\vec{r})\, e^{-in\omega t + in\beta_e Y}. \qquad (1)$$

Here $\beta_e \equiv \omega/v_0$ is the electron wave number with ω being the modulation frequency, $\vec{r}$ denotes the spatial coordinates. The expression (1) means we consider the constant component of the beam charge density ρ_0 compensated by positive ions, which usually occurs in the beam. By using the charge conservation law and assuming unidirectional motion of the beam particles along the Y-axis (due to the infinite longitudinal static magnetic field, for example) (see Fig. 1), the harmonic amplitudes can be expressed as [4]

$$j_n = I_0 i_n(Y)\sigma(y),$$

$$\rho_n = \frac{I_0}{v_0} i_n(Y)\sigma(y),$$

where I_0 is the total beam current, $i_n = \frac{1}{2\pi}\int_0^{2\pi} e^{in(\varphi_0+\Theta)}\mathrm{d}\varphi_0$ is the n-th harmonic of the beam current normalized on I_0; φ_0 is the unperturbed electron phase, and θ is the electron phase shift due to the both microwave and Coulomb field action. By $\sigma(y)$ we denote the y-projection of the beam current density transversal distribution. Such a factorization is possible because of uniformity of the wave supporting system along the y-axis. In general, a component of the space charge field acting on the beam particles in a beam modulated according to (1) can be expressed as follows [4]:

$$E_q = \int_V \sum_{n=1}^{\infty} \rho_n(y',Y')\, e^{-in\omega t + in\beta_e Y' + in\beta_e y'} \times$$
$$\times G(y,y',Y,Y')\,\mathrm{d}y'\mathrm{d}Y',$$

where V is the volume occupied by the beam, and $G(y,y',Y,Y')$ is the Green's function for the electric field equation. The aim of this paper is to derive an expression for the space charge field acting on the beam particles, which would be easy for using in theoretical studies and time saving in computer simulations.

3. Space Charge Field

For more clarity, let us consider the beam modulated by an arbitrary n-th time harmonic only. As the tilt angle is usually small, further we assume $\cos\alpha \cong 1$. In this terms

$$E_{qY} = -\sin\alpha \frac{I_0}{v_0} e^{-in\omega t}\int_{Y_i}^{0}\mathrm{d}Y' i_n(Y') \times$$
$$\times \int_{-\Delta/2}^{\Delta/2}\mathrm{d}y'\sigma(y')\, e^{in\beta_e(y'+Y')} G(y,Y,y',Y'), \qquad (2)$$

where $\Delta \equiv a/\sin\alpha$ is the area on the lower plane, covered by the beam (see Fig. 1), and Y_i means the Y-coordinate of the injection plane; further we assume $Y_i = -D/\sin\alpha$. To evaluate (2), let us make use of the Taylor expansion of the current harmonic amplitude $i_n(Y')$ in the vicinity of the observation point Y [2]:

$$i_n(Y') = i_n(Y) + \sum_{m=1}^{\infty}\frac{1}{m!}\frac{\mathrm{d}^m i_n}{\mathrm{d}Y^m}(Y'-Y)^m.$$

Now we may rewrite the expression (2) for the space charge field as below:

$$E_{qY} = -\sin\alpha \frac{I_0}{v_0} e^{-in\omega t} \times$$
$$\times\Bigg\{ i_n(Y)\int_{Y_i}^{0}\mathrm{d}Y'\int_{-\Delta/2}^{\Delta/2}\mathrm{d}y'\sigma(y')\, e^{in\beta_e(y'+Y')}G(y,Y,y',Y') +$$
$$+ \sum_{m=1}^{\infty}\frac{1}{m!}\frac{\mathrm{d}^m i_n}{\mathrm{d}Y^m}\int_{Y_i}^{0}\mathrm{d}Y'(Y'-Y)^m \times$$
$$\times \int_{-\Delta/2}^{\Delta/2}\mathrm{d}y'\sigma(y')\, e^{in\beta_e(y'+Y')}G(y,Y,y',Y')\Bigg\}. \qquad (3)$$

The first term in the braces is responsible for local effect of space charge, whereas the second term describes non-local phenomena. As $i_n(Y')$ varies slowly as compared to the Green's function, the main contribution to (3) is due to the local action, so let us first neglect non-local phenomena and consider only the first term in (3).

3.1. Local Phenomena

The Green's function (A1), derived in Appendix, can be rewritten as a function of y and Y as follows: $G = G^+ - G^-$. Here

$$G^{\pm} = -\frac{\sinh\frac{\pi}{D}\left[(y-y')+(Y-Y')\right]}{8\varepsilon_0 D \sinh\frac{\pi\chi_-^{\pm}}{2D}\sinh\frac{\pi\chi_+^{\pm}}{2D}},$$

where $\chi_+^{\pm} \equiv (y-y')+(Y-Y')+i(Y\pm Y')\sin\alpha$, $\chi_-^{\pm} \equiv (y-y')+(Y-Y')-i(Y\pm Y')\sin\alpha$, ε_0 is the permittivity of a free space.

In order to simplify the expression (3) for the space charge field let us consider the integral

$$I_a^{\pm} = \int_{-\Delta/2}^{\Delta/2} \mathrm{d}y'\sigma(y')e^{in\beta_e y'}\frac{\sinh\frac{\pi}{D}\left[(y-y')+(Y-Y')\right]}{\sinh\frac{\pi\chi_-^{\pm}}{2D}\sinh\frac{\pi\chi_+^{\pm}}{2D}} \qquad (4)$$

as distinct from

$$I_\infty^{\pm}(n\beta_e) = \int_{-\infty}^{\infty} \mathrm{d}y' e^{in\beta_e y'}\frac{\sinh\frac{\pi}{D}\left[(y-y')+(Y-Y')\right]}{\sinh\frac{\pi\chi_-^{\pm}}{2D}\sinh\frac{\pi\chi_+^{\pm}}{2D}}. \qquad (5)$$

Now we note that $I_\infty^{\pm}$ is the Fourier spectrum of the function $-8\varepsilon_0 D\cdot G^{\pm}(y,Y,y',Y')$, whereas $I_a^{\pm}$ is the spectrum of the product $-8\varepsilon_0 D\cdot G^{\pm}(y,Y,y',Y')\cdot\sigma(y')$, which is the convolution of the multipliers spectra. So, we can write

$$I_a^{\pm} = F\left[\sigma(y')\cdot\left(-8\varepsilon_0 D\cdot G^{\pm}\right)\right] = \left(F[\sigma(y')] * I_\infty^{\pm}\right), \quad (6)$$

where $F[f(x)] \equiv \int_{-\infty}^{\infty} f(x)e^{in\beta_e x}\mathrm{d}x$, and $(f_1(t) * f_2(t)) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)\mathrm{d}\tau$. Denoting $F[\sigma(y')] \equiv \Psi(n\beta_e)$, we can express the examined integral as follows:

$$I_a^{\pm} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}k\Psi(n\beta_e - k)I_\infty^{\pm}(k)$$

Thus, the integral (4) describing the Coulomb field in a beam with any transversal current distribution can be expressed through the Coulomb force in the infinitely broad beam with a homogeneous distribution of harmonics amplitudes (equations (5), (6)).

Now let us brace ourselves to evaluating $I_\infty^{\pm}$. This can be done using the residue technique. Considering y' a complex number, one notes that at $y' \to \pm\infty$ and at $y' \to +i\infty$ the function $e^{i\beta_e y'}G(y,y',Y,Y')$ vanishes as exponentially; so Jordan's lemma's conditions are met and the integrals can be expressed through the residues of the integrand in the upper half-plane of the complex plane. Let us first examine I_∞^{+}. In the upper half-plane the integrand has only first-order poles at points

$$\xi_n^- = y+(Y-Y')-i(Y+Y')\sin\alpha+2Dni, \quad n>0$$

$$\xi_m^+ = y+(Y-Y')+i(Y+Y')\sin\alpha+2Dmi, \quad m>0.$$

Taking into account that for any $s > r \geq 0$ the ratio of residues (noted as Res) of the intergrand in (5) $\underset{\xi_s^{\pm}}{Res}\Big/\underset{\xi_r^{\pm}}{Res}$ is not larger than $e^{-n\beta_e D}$ and assuming $\beta_e D >> 1$, we may approximate I_∞^{+} as

$$I_\infty^{+} \cong 2\eth i\left(\underset{\xi_0^-}{Res}+\underset{\xi_1^+}{Res}\right) = -4iDe^{in\beta_e(y+Y)}\times$$
$$\times\left[e^{n\beta_e\sin\alpha(Y+Y')}+e^{-2n\hat{a}_e D}e^{-n\hat{a}_e\sin\alpha(Y+Y')}\right]e^{-in\hat{a}_e Y'}.$$

Similarly, one obtains

$$I_\infty^{-} = -4iDe^{in\beta_e(y+Y)}\times$$
$$\times\left[e^{-n\beta_e\sin\alpha|Y-Y'|}+e^{-2n\beta_e D}e^{n\beta_e\sin\alpha|Y-Y'|}\right]e^{-in\beta_e Y'}$$

(considering separately the cases $Y > Y'$ and $Y < Y'$). Also, allowing $\beta_e \in \Re$ and considering separately the cases $\beta_e > 0$ and $\beta_e < 0$, it is easy to generalize the expression for $I_\infty^\pm$ as follows:

$$I_\infty^\pm = -4iD\,\mathrm{sgn}(\beta_e)\, e^{in\beta_e(y+Y)} \times$$
$$\times\left[e^{-n|\beta_e|\sin\alpha|Y\pm Y'|} + e^{-n|\beta_e|(2D-\sin\alpha|Y\pm Y'|)}\right]e^{-in\beta_e Y'}. \quad (7)$$

This expression can be made more precise (to a desirable extent) by taking into account more poles of the infinite set.

With the expression (7) the integral $I_a^\pm$ becomes

$$I_a^\pm = -\frac{2iD}{\pi}\int_{-\infty}^{\infty} \mathrm{d}k\,\mathrm{sgn}(k)\Psi(n\beta_e - k)\times$$
$$\times e^{ikY} e^{i|k|y} e^{-ikY'}\left[e^{-n|k|\sin\alpha|Y\pm Y'|} + e^{-n|k|(2D-\sin\alpha|Y\pm Y'|)}\right].$$

Integrating the latter expression by Y' and taking into account (4), we can expand the space charge field (3) as follows, neglecting the terms describing non-local effects of the field:

$$E_{qY} = \frac{e^{-in\omega t} i_n(Y)\sin^2\alpha}{2\pi i\varepsilon_o v_0}\times$$
$$\times\int_{-\infty}^{\infty}\mathrm{d}k\frac{k}{k^2\sin^2\alpha + (n\beta_e - k)^2}\Psi(n\beta_e - k)\times$$
$$\times\left[\left(e^{|k|\sin\alpha Y} - e^{i(n\beta_e - k)}\right) - 2e^{-|k|D}e^{-i(n\beta_e-k)D/\sin\alpha}\times\right.$$
$$\left.\times\sinh(|k|\sin\alpha Y)\right]e^{ik(y+Y)} =$$
$$= -\frac{\omega_p^2 m_e}{|e|n\beta_e}\frac{i_n(Y)}{i}e^{-i(\omega t - n\beta_e y - n\beta_e Y)}R_{n0}^2(y,Y),$$

where:

$$R_{n0}^2(y,Y) = -\frac{n\beta_e\sin^2\alpha}{2\pi}\times$$
$$\times\int_{-\infty}^{\infty}\mathrm{d}k\frac{k}{k^2\sin^2\alpha - (n\beta_e-k)^2}\Psi(n\beta_e - k)\times$$
$$\times\left[\left(e^{|k|\sin\alpha Y} - e^{i(n\beta_e-k)Y}\right) - 2e^{-|k|D}\times\right.$$
$$\left.\times e^{-i(n\beta_e-k)D/\sin\alpha}\sinh(|k|\sin\alpha Y)\right]e^{i(k-n\beta_e)(y+Y)} \quad (8)$$

is the space charge field depression coefficient for an arbitrary beam current density distribution.

When applying the depression coefficients technique to describing collective electron interaction regimes of electron devices with an inclined beam, it is useful to introduce the averaged over a cross-section Y=const depression coefficient, taking into account that the energy delivered (or absorbed) by the beam is the sum of the energies delivered (absorbed) by the beam layers:

$$\overline{R}_{n0}^2(Y) = -\frac{an\beta_e\sin\alpha}{4\pi}\times$$
$$\times\int_{-\infty}^{\infty}\mathrm{d}k\frac{k}{k^2\sin^2\alpha + (n\beta_e-k)^2}\frac{\sin x}{x}\Psi(n\beta_e - k)\times$$
$$\times\left[\left(e^{|k|\sin\alpha Y} - e^{i(n\beta_e-k)Y}\right) - 2e^{-|k|D}e^{-i(n\beta_e-k)D/\sin\alpha}\times\right.$$
$$\left.\times\sinh(|k|\sin\alpha Y)\right]e^{i(k-n\beta_e)Y}, \quad (9)$$

where $x \equiv (k - \beta_e)a/2\sin\alpha$. Let us note here that, substituting in (8), (9) the spectrum of any transversal current density distribution, we will be able to compute the space charge field in that case. For example, for a homogeneous distribution of the current density

$$\sigma(y') \equiv \begin{cases} 1/S, y' \in [-\Delta/2, \Delta/2], \\ 0, y' \notin [-\Delta/2, \Delta/2]; \end{cases}$$

where S is the beam cross-section, which is assumed to be equal unity. It is easy to show that

$$F[\sigma(y')] = \frac{e^{in\beta_e\Delta/2} - e^{-in\beta_e\Delta/2}}{in\beta_e}.$$

The depression coefficients (8), (9) become

$$R_{n0}^2(y,Y) = -\frac{n\beta_e\sin^2\alpha}{\pi}\times$$
$$\times\int_{-\infty}^{\infty}\mathrm{d}k\frac{k}{k^2\sin^2\alpha - (n\beta_e-k)^2}\sin\frac{(n\beta_e-k)a}{2\sin\alpha}(n\beta_e-k)^{-1}\times$$
$$\times\left[\left(e^{|k|\sin\alpha Y} - e^{i(n\beta_e-k)Y}\right) - 2e^{-|k|D}e^{-i(n\beta_e-k)D/\sin\alpha}\sinh(|k|\sin\alpha Y)\right]\times$$
$$\times e^{i(k-n\beta_e)(y+Y)}, \quad (10)$$

$$\overline{R}_{n0}^2(Y) = \frac{an\beta_e\sin\alpha}{2\pi}\times$$
$$\times\int_{-\infty}^{\infty}\mathrm{d}k\frac{k}{k^2\sin^2\alpha + (n\beta_e-k)^2}\left(\frac{\sin x}{x}\right)^2\times$$
$$\times\left[\left(e^{|k|\sin\alpha Y} - e^{i(n\beta_e-k)}\right) - 2e^{-|k|D}e^{-i(n\beta_e-k)D/\sin\alpha}\times\right.$$
$$\left.\times\sinh(|k|\sin\alpha Y)\right]e^{i(k-n\beta_e)Y} \quad (10')$$

The plasma frequency reduction factor R_{10} (10) is shown in Fig. 2 as a function of y and $z \equiv -Y \sin\alpha$. In general, the integrals in (10) cannot be evaluated in a closed form. However, analytical expressions can be derived for two important cases: $\beta_e a >> 1$ is a broad beam and $\beta_e a \to 0$ is a narrow beam. In the case of a broad beam we note that the integrand in (10)

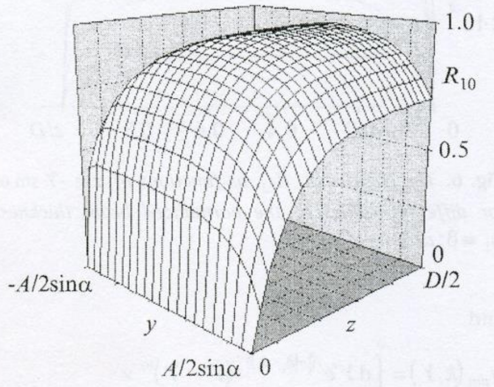

Fig. 2. *The plasma frequency reduction factor R_{10} as a function of y and $z \equiv -Y \sin\alpha$, $\beta_e a = 5$*

includes the function $\frac{k}{k^2 \sin^2\alpha + (n\beta_e - k)^2}\left(\frac{\sin x}{x}\right)$ constituting a sharp peak in the vicinity of the point $k = n\beta_e$, the other terms being slow functions of k. Using the Taylor expansion of the slow terms in the vicinity of $k = n\beta_e$ and taking into account only the first term of the expansion, we can approximate (10) as follows:

$$R_{n0}^2(y,Y) \approx \left[\left(1 - e^{n\beta_e \sin\alpha Y}\right) + 2e^{-n\beta_e D}\sinh(n\beta_e \sin\alpha Y)\right] \cdot K_n(a,y), \qquad (11)$$

where

$$K_n(a,y) = 1 - e^{-an\beta_e/2}\cosh(yn\beta_e \sin\alpha) \qquad (12)$$

is a function describing the effect of the finite beam thickness on the space charge depression coefficient. It is easy to see that for the depression coefficient averaged over y instead of (12) we will have

$$\overline{K}_n(a) = 1 - \frac{2e^{-an\beta_e/2}}{an\beta_e}\sinh\frac{an\beta_e}{2}. \qquad (13)$$

In the case of a narrow beam, in the integral (10) we can assume $\sin x / x \cong 1$. This leads to the following expression for the depression coefficient.

$$\overline{R}_{n0}^2 = 1 - e^{2n\beta_e \sin\alpha Y} + 2e^{-2n\beta_e D} \times \sinh(n\beta_e \sin\alpha Y)e^{-n\beta_e \sin\alpha Y} \qquad (14)$$

Fig. 3, 4 show the averaged plasma frequency reduction factors $\overline{R}_{10}$ vs. the z-coordinate for different values of the beam thickness for the cases of a thick and a thin beams, respectively, as computed according to (10) (solid lines) and (11)-(14)

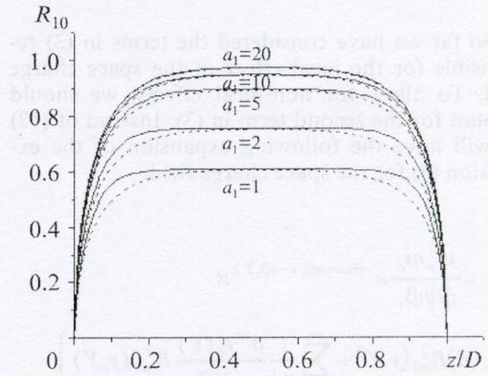

Fig. 3. *The averaged plasma frequency reduction factor $\overline{R}_{10}$ vs. the z-coordinate for different values of the normalized beam thickness $a_1 = a\beta_e$ for a thick beam*

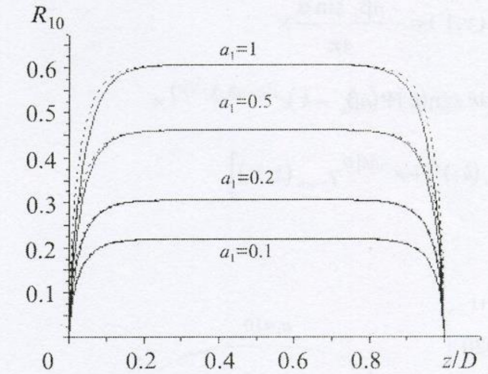

Fig. 4. *The averaged plasma frequency reduction factors $\overline{R}_{10}$ vs. the z-coordinate for different values of the normalised beam thickness $a_1 = a\beta_e$ for a thin beam*

(dotted lines). It is evident from Fig. 3 that the analytical expression (11) serves a good approximation of (10) for $\beta_e a \ge 2$, whereas the formula (14) works for $\beta_e a \le 1$. It can be seen from (12), (13) that the effect of the beam edges on the space charge field is determined, as it should be, by the phase shift $\beta_e a$, not by the beam thickness itself, and for $\beta_e a \ge 10$ can be neglected.

As one could expect, the reduction coefficient diminishes essentially at distance from the metallic surfaces of the order of β_e^{-1}. It follows from here that one may neglect mutual influence of the planes if the distance between the metallic planes is larger than this value.

3.2. Non-local Phenomena

So far we have considered the terms in (3) responsible for the local effect of the space charge field. To allow for non-local effects, we should account for the second term in (3). Instead of (12) we will have the following expansion of the expression (3) for the space charge field:

$$E_{qY} = \frac{\omega_p^2 m_e}{i|e|n\beta_e} e^{-i(\omega t - n\beta_e y - n\beta_e Y)} \times$$
$$\times\left(i_n(Y) R_{n0}^2(y,Y) + \sum_{m=1}^{\infty} \frac{1}{m!} \frac{\mathrm{d}^m i_n(Y)}{\mathrm{d}Y^m} R_{nm}^2(y,Y) \right) \quad (15)$$

where ω_p is the plasma frequency;

$$R_{nm}^2(y,Y) = -\frac{n\beta_e \sin\alpha}{4\pi} \times$$
$$\times \int_{-\infty}^{\infty} \mathrm{d}k\, \mathrm{sgn}(k) \Psi(n\beta_e - k) e^{i(k - n\beta_e)(y+Y)} \times$$
$$\left[\gamma_{nm}(k,Y) + e^{-2|k|D} \gamma_{-nm}(k,Y) \right],$$

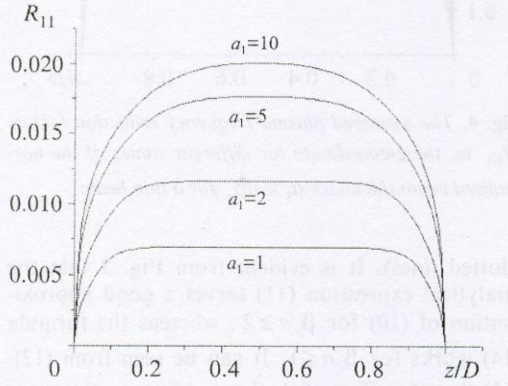

Fig. 5. *The coefficient* $\overline{R}_{11}$ *as functions of* $z \equiv -Y \sin\alpha$ *for different values of the normalized beam thickness* $a_1 \equiv \beta_e a$

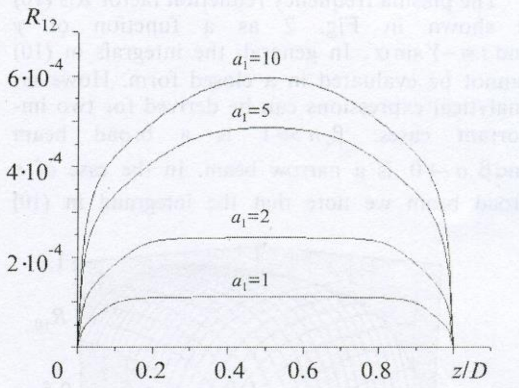

Fig. 6. *The coefficient* $\overline{R}_{12}$ *as functions of* $z \equiv -Y \sin\alpha$ *for different values of the normalized beam thickness* $a_1 \equiv \beta_e a$

and

$$\gamma_{nm}(k,Y) = \int_{Y_1}^{0} \mathrm{d}Y' e^{i(|n|\beta_e - k)Y'} (Y' - Y)^m \times$$
$$\times\left(e^{n|k|\sin\alpha(Y+Y')} - e^{-n|k|\sin\alpha|Y-Y'|} \right).$$

The coefficients $\overline{R}_{11}$ and $\overline{R}_{12}$ averaged over y are shown in Fig. 5, 6 as functions of $z \equiv -Y \sin\alpha$ for different values of the normalized beam thickness $a_1 \equiv \beta_e a$. With a_1 increasing, the coefficients for non-local effect turn to their limit values. These values rapidly decrease with m growth. However, even the maximum value of R_{11} is essentially smaller than that of R_{10}. Because of this fact, the non-local phenomena are not crucial for the space charge effects description and in most cases it is sufficient to account only for the first term in the sum over m in (15) (see also [3]).

4. Conclusions

In this paper, we proposed an approach to computing the space charge field in a modulated beam inclined to the principal plane of the wave supporting system, that enabled us to obtain the simplified analytical expressions for the space charge field by using space charge depression coefficients. The approach proposed gives an elegant and flexible method for accounting for effect of the geometry of the metallic surfaces and the beam on the space charge field. The derived expressions reduce the problem of calculating space charge field for a given geometry to computing the Coulomb force in a beam in free space with weight coefficients depending on spatial coordinates, which we were able to evaluate analytically. We analyzed both local and non-local action of space charge and their comparative contribution to the quasi-static field. The results obtained for a ribbon-like beam can be generalized for an arbitrary transversal distribution of the beam current density.

Acknowledgements

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Appendix.

1. The Green's Function for a Narrow Rectangular Resonator

In order to derive the Green's function for the geometry shown in Fig. 1, we will issue from the calculation of the Green's function for a narrow rectangular resonator. This will allow us to obtain conditions at which the rectangular cavity can be modelled by two infinite metallic planes. To this end we regard the field created by a point unitary charge whose coordinates as well as those of the point of observation we consider components of complex numbers. Let $z=x+iy$ be the point observation and $\zeta=\xi+i\eta$ the point where the charge is located, (x,y), (ξ,η) their Cartesian coordinates, respectively. Let $x,\xi\in[0,D]$, $y,\eta\in[0,L]$, where $L>>D$.

The force function of the potential field can be expressed through Weierstrass's functions as follows [6]:

$$U(z)=2Re\ln\frac{\sigma(z+\bar{\zeta})\sigma(z-\bar{\zeta})}{\sigma(z+\zeta)\sigma(z-\zeta)},\qquad \sigma(z)=\text{const},$$

$$e^{\delta z^2/2\tau}\theta_1\left(\frac{z}{\tau}\right),\qquad \tau=2D,$$

where dash means conjunction and θ_1 is Jakoby's function [6] (see below). The potential becomes

$$U(z)=2Re\times$$
$$\times\ln\left[e^{\frac{\delta}{2\tau}\left((z+\bar{\zeta})^2+(z-\bar{\zeta})^2-(z+\zeta)^2-(z-\zeta)^2\right)}\frac{\theta_1\left((z+\bar{\zeta})/\tau\right)\theta_1\left((z-\bar{\zeta})/\tau\right)}{\theta_1\left((z+\zeta)/\tau\right)\theta_1\left((z-\zeta)/\tau\right)}\right]=$$
$$=2Re\left\{-\frac{\delta}{2\tau}8i\xi\eta+\ln\frac{\theta_1\left((z+\bar{\zeta})/\tau\right)\theta_1\left((z-\bar{\zeta})/\tau\right)}{\theta_1\left((z+\zeta)/\tau\right)\theta_1\left((z-\zeta)/\tau\right)}\right\}.$$

Keeping in mind that the Green's function is expressed through the derivative $\partial U/\partial y$, we can omit the terms constant with respect to y and write

$$U(z)=2Re\ln\ln\frac{\theta_1\left((z+\bar{\zeta})/\tau\right)\theta_1\left((z-\bar{\zeta})/\tau\right)}{\theta_1\left((z+\zeta)/\tau\right)\theta_1\left((z-\zeta)/\tau\right)}$$

Now let us take advantage of the assumption that $L>>D$:

$$\theta_1(z)=i\sum_{k=-\infty}^{\infty}(-1)^k\left(e^{-\pi L/D}\right)^{(k-1/2)^2}e^{(2k-1)\pi iz}\equiv$$
$$\cong 2e^{-\frac{\pi L}{4D}}\sin\pi z-2e^{-\frac{9\pi L}{4D}}\sin 3\pi z.$$

Here we neglected the terms at least by the factor $e^{4\pi L/D}$ less then those remained. With this

$$U(z)=2Re\times$$
$$\times\ln\left[\frac{\sin\frac{\pi}{\tau}(z+\bar{\zeta})\sin\frac{\pi}{\tau}(z-\bar{\zeta})}{\sin\frac{\pi}{\tau}(z+\zeta)\sin\frac{\pi}{\tau}(z-\zeta)}\left(1+O\left(e^{-2\pi L/D}\right)\right)\right]$$

and, after some transformations, one obtains

$$U(z)\approx\ln\left[\frac{\sin^2\frac{\pi}{2D}(x+\xi)+\sinh^2\frac{\pi}{2D}(y-\eta)}{\sin^2\frac{\pi}{2D}(x-\xi)+\sinh^2\frac{\pi}{2D}(y-\eta)}\times\right.$$
$$\left.\times\left(1+O\left(e^{-2\pi L/D}\right)+O\left(e^{-\frac{\pi}{2D}(y+\eta)}\right)+O\left(e^{-\frac{\pi}{2D}(2L-y-\eta)}\right)\right)\right].$$

Differentiating the latter expression we come out with the final expression for the Green's function:

$$G\equiv\frac{1}{4\pi\varepsilon_0}\frac{\partial U}{\partial y}=$$
$$=-\frac{\pi}{4\pi\varepsilon_0 D}\sinh\frac{\pi}{D}(y-\eta)\sin\frac{\pi}{D}x\sin\frac{\pi}{D}\xi\times$$
$$\times\left[\sin^2\frac{\pi}{2D}(x+\xi)+\sinh^2\frac{\pi}{2D}(y-\eta)\right]^{-1}\times$$
$$\times\left[\sin^2\frac{\pi}{2D}(x-\xi)+\sinh^2\frac{\pi}{2D}(y-\eta)\right]^{-1}\times$$
$$\times\left(1+O\left(e^{-2\pi L/D}\right)+O\left(e^{-\frac{\pi}{2D}(y+\eta)}\right)+O\left(e^{-\frac{\pi}{2D}(2L-y-\eta)}\right)\right).\qquad\text{(A1)}$$

The last line of this expression shows the estimates of the terms neglected. In particular, it follows from (A1) that it is possible for a narrow rectangular resonator to be modelled by two infinite planes when $e^{2\pi L/D} >> 1$, and the charge particles being at a distance more than D from the resonator's sidewalls. These conditions are not severe from the point of view of practical configurations.

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Коэффициенты редукции плазменной частоты при рассеянии электронного пучка на металлических поверхностях

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Метод коэффициентов депрессии пространственного заряда обобщен на случай распространения ленточного электронного пучка с произвольным распределением тока под углом к металлической поверхности. Получены и проанализированы выражения для коэффициентов депрессии. Рассмотрены локальное и нелокальное действие пространственного заряда и их сравнительный вклад в квазистатическое поле.

Коефіцієнти редукції плазмової частоти для розсіювання електронного пучка на металевих поверхнях

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Метод коефіцієнтів депресії просторового заряду узагальнено на випадок розповсюдження стрічкового електронного пучка з довільним розподіленням струму під кутом до металевої поверхні. Отримано та проаналізовано вирази для коефіцієнтів депресії. Розглянуто локальну та нелокальну дію просторового заряду та їх порівняльний внесок у квазістатичне поле.